

Least squares II — fitting a line to data (regression!)

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Practice problems

1. Find the least squares line $y = mx + c$ for the points $(1, 2), (2, 3), (3, 5)$.
2. Given the points $(0, 1), (1, 1), (2, 1)$, what is the least squares line?
3. If we have n points and we force the line to pass through the origin ($c = 0$), how does the matrix A change?

Solutions

1. Find the least squares line $y = mx + c$ for the points $(1, 2), (2, 3), (3, 5)$.

- Construct $A = \begin{pmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{pmatrix}$ and $b = \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}$.
- Compute $A^T A = \begin{pmatrix} 1+4+9 & 1+2+3 \\ 1+2+3 & 1+1+1 \end{pmatrix} = \begin{pmatrix} 14 & 6 \\ 6 & 3 \end{pmatrix}$.
- Compute $A^T b = \begin{pmatrix} 1(2) + 2(3) + 3(5) \\ 2 + 3 + 5 \end{pmatrix} = \begin{pmatrix} 23 \\ 10 \end{pmatrix}$.
- Solve $\begin{pmatrix} 14 & 6 \\ 6 & 3 \end{pmatrix} \begin{pmatrix} m \\ c \end{pmatrix} = \begin{pmatrix} 23 \\ 10 \end{pmatrix}$.
- From the second equation, $3c = 10 - 6m$, so $c = 10/3 - 2m$.
- Substitute into first: $14m + 6(10/3 - 2m) = 23 \implies 14m + 20 - 12m = 23 \implies 2m = 3 \implies m = 1.5$.
- Then $c = 10/3 - 3 = 1/3$.

Answer: $y = 1.5x + 1/3$

2. Given the points $(0, 1), (1, 1), (2, 1)$, what is the least squares line?

- The points are horizontal. The line $y = 1$ should be the best fit.
- Construct $A = \begin{pmatrix} 0 & 1 \\ 1 & 1 \\ 2 & 1 \end{pmatrix}$ and $b = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.
- Compute $A^T A = \begin{pmatrix} 5 & 3 \\ 3 & 3 \end{pmatrix}$.
- Compute $A^T b = \begin{pmatrix} 0+1+2 \\ 1+1+1 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$.
- Solve $\begin{pmatrix} 5 & 3 \\ 3 & 3 \end{pmatrix} \begin{pmatrix} m \\ c \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$.
- Subtracting equations gives $2m = 0 \implies m = 0$.
- Then $3(0) + 3c = 3 \implies c = 1$.

Answer: $y = 1$

3. If we have n points and we force the line to pass through the origin ($c = 0$), how does the matrix A change?

- The equation becomes $y = mx$.
- The system is now $mx_i = y_i$ for each i .
- The matrix A no longer needs the column of ones.
- A becomes a single column vector containing the x values: $A = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$.

Answer: A becomes a single column vector of x -coordinates.