

Projection onto a subspace — the projection matrix

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Practice problems

1. Find the projection matrix P for the subspace spanned by the vector $a = [1, 1]^T$.
2. Given the projection matrix $P = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, project the vector $b = [3, 7]^T$ onto the subspace.
3. Verify if the matrix $M = \begin{bmatrix} 0.6 & 0.8 \\ 0.8 & 0.6 \end{bmatrix}$ is a projection matrix.

Solutions

1. Find the projection matrix P for the subspace spanned by the vector $a = [1, 1]^T$.

- Set $A = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.
- Compute $A^T A = [1 \ 1] \begin{bmatrix} 1 \\ 1 \end{bmatrix} = [2]$.
- The inverse is $(A^T A)^{-1} = [1/2]$.
- Compute $P = A(A^T A)^{-1} A^T = \begin{bmatrix} 1 \\ 1 \end{bmatrix} [1/2] [1 \ 1] = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}$.

Answer: $P = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}$

2. Given the projection matrix $P = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, project the vector $b = [3, 7]^T$ onto the subspace.

- The projection is $p = Pb$.
- Compute $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 7 \end{bmatrix} = \begin{bmatrix} 1(3) + 0(7) \\ 0(3) + 0(7) \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$.

Answer: $p = [3, 0]^T$

3. Verify if the matrix $M = \begin{bmatrix} 0.6 & 0.8 \\ 0.8 & 0.6 \end{bmatrix}$ is a projection matrix.

- A projection matrix must satisfy $M^2 = M$.
- Compute $M^2 = \begin{bmatrix} 0.6 & 0.8 \\ 0.8 & 0.6 \end{bmatrix} \begin{bmatrix} 0.6 & 0.8 \\ 0.8 & 0.6 \end{bmatrix}$.
- Top left: $0.6(0.6) + 0.8(0.8) = 0.36 + 0.64 = 1.0$.
- Since the top left entry of M^2 is 1.0 and the top left entry of M is 0.6, $M^2 \neq M$.

Answer: No, it is not a projection matrix.