

Projection onto a line

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Practice problems

1. Find the projection of $b = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$ onto $a = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$.
2. Project the vector $b = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ onto the vector $a = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$.
3. Given $a = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $b = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$, find the projection p and the error vector e . Verify that $p \cdot e = 0$.

Solutions

1. Find the projection of $b = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$ onto $a = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$.

- Compute $a \cdot b = (3)(5) + (4)(2) = 15 + 8 = 23$.
- Compute $a \cdot a = 3^2 + 4^2 = 9 + 16 = 25$.
- Calculate the scalar $c = 23/25 = 0.92$.
- Multiply c by a : $0.92 \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 2.76 \\ 3.68 \end{bmatrix}$.

Answer: $\begin{bmatrix} 2.76 \\ 3.68 \end{bmatrix}$

2. Project the vector $b = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ onto the vector $a = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$.

- Compute $a \cdot b = (0)(1) + (1)(1) + (1)(0) = 1$.
- Compute $a \cdot a = 0^2 + 1^2 + 1^2 = 2$.
- The scalar is $c = 1/2$.
- The projection is $p = \frac{1}{2} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0.5 \\ 0.5 \end{bmatrix}$.

Answer: $\begin{bmatrix} 0 \\ 0.5 \\ 0.5 \end{bmatrix}$

3. Given $a = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $b = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$, find the projection p and the error vector e . Verify that $p \cdot e = 0$.

- Compute $a \cdot b = (1)(-1) + (2)(1) = 1$.
- Compute $a \cdot a = 1^2 + 2^2 = 5$.
- Find $p = \frac{1}{5} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0.2 \\ 0.4 \end{bmatrix}$.
- Find $e = b - p = \begin{bmatrix} -1 - 0.2 \\ 1 - 0.4 \end{bmatrix} = \begin{bmatrix} -1.2 \\ 0.6 \end{bmatrix}$.
- Check $p \cdot e = (0.2)(-1.2) + (0.4)(0.6) = -0.24 + 0.24 = 0$.

Answer: $p = \begin{bmatrix} 0.2 \\ 0.4 \end{bmatrix}$, $e = \begin{bmatrix} -1.2 \\ 0.6 \end{bmatrix}$