

Transformation clinic — identify the matrix from the picture

Lesson 81 · Module 5 — Linear Transformations · Linear Algebra · dailymaths.uk/linalg

Practice problems

1. A linear transformation maps the basis vector $e_1 = [1, 0]^T$ to $[2, 0]^T$ and the basis vector $e_2 = [0, 1]^T$ to $[1, 1]^T$. Find the standard matrix A .
2. A transformation is seen to rotate the plane by 90° counter-clockwise and then scale the entire plane by a factor of 3. What is the resulting matrix A ?
3. A transformation collapses the plane onto the line $y = -x$. The image of e_1 is $[1, -1]^T$ and the image of e_2 is $[-2, 2]^T$. Find the matrix A and verify its determinant.

Solutions

1. A linear transformation maps the basis vector $e_1 = [1, 0]^T$ to $[2, 0]^T$ and the basis vector $e_2 = [0, 1]^T$ to $[1, 1]^T$. Find the standard matrix A .

- The first column of the matrix is the image of e_1 , which is $[2, 0]^T$.
- The second column of the matrix is the image of e_2 , which is $[1, 1]^T$.
- Placing these into a matrix gives $A = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$.

Answer: $\begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$

2. A transformation is seen to rotate the plane by 90° counter-clockwise and then scale the entire plane by a factor of 3. What is the resulting matrix A ?

- A 90° rotation maps $[1, 0]^T$ to $[0, 1]^T$ and $[0, 1]^T$ to $[-1, 0]^T$.
- Scaling by 3 multiplies these resulting vectors by 3.
- The new image of e_1 is $[0, 3]^T$ and the new image of e_2 is $[-3, 0]^T$.
- The matrix is $A = \begin{bmatrix} 0 & -3 \\ 3 & 0 \end{bmatrix}$.

Answer: $\begin{bmatrix} 0 & -3 \\ 3 & 0 \end{bmatrix}$

3. A transformation collapses the plane onto the line $y = -x$. The image of e_1 is $[1, -1]^T$ and the image of e_2 is $[-2, 2]^T$. Find the matrix A and verify its determinant.

- The first column is $T(e_1) = [1, -1]^T$.
- The second column is $T(e_2) = [-2, 2]^T$.
- The matrix is $A = \begin{bmatrix} 1 & -2 \\ -1 & 2 \end{bmatrix}$.
- The determinant is $(1)(2) - (-2)(-1) = 2 - 2 = 0$.

Answer: $\begin{bmatrix} 1 & -2 \\ -1 & 2 \end{bmatrix}$