

Why similar matrices represent the same map

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Practice problems

1. Given $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$, determine if they could be similar by comparing their determinants and traces.
2. If matrix A has $\det(A) = 5$ and $\operatorname{tr}(A) = 2$, and $B = P^{-1}AP$, what are $\det(B)$ and $\operatorname{tr}(B)$?
3. Determine if $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is similar to $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$.

Solutions

1. Given $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$, determine if they could be similar by comparing their determinants and traces.

- Compute $\det(A) = (2)(2) - (1)(1) = 3$.
- Compute $\det(B) = (3)(1) - (0)(0) = 3$.
- Compute $\text{tr}(A) = 2 + 2 = 4$.
- Compute $\text{tr}(B) = 3 + 1 = 4$.
- Since both the determinant and trace match, the matrices can be similar.

Answer: Yes, they share the same invariants ($\det = 3, \text{tr} = 4$).

2. If matrix A has $\det(A) = 5$ and $\text{tr}(A) = 2$, and $B = P^{-1}AP$, what are $\det(B)$ and $\text{tr}(B)$?
- Recall that the determinant is invariant under similarity, so $\det(B) = \det(A) = 5$.
 - Recall that the trace is invariant under similarity, so $\text{tr}(B) = \text{tr}(A) = 2$.

Answer: $\det(B) = 5, \text{tr}(B) = 2$

3. Determine if $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is similar to $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$.

- Compute $\det(A) = 1$ and $\det(B) = 1$.
- Compute $\text{tr}(A) = 2$ and $\text{tr}(B) = 2$.
- The invariants match, but similarity requires $B = P^{-1}AP$.
- For $A = I$, $P^{-1}IP = P^{-1}P = I$.
- Since $B \neq I$, A and B cannot be similar despite having the same invariants.

Answer: No, because the only matrix similar to the identity matrix is the identity matrix itself.