

Composition of transformations = matrix multiplication (revisited)

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Practice problems

1. Find the standard matrix C for the composition of a reflection across the y -axis followed by a scaling of the x -axis by a factor of 5.
2. Let R be a rotation by 90° counter-clockwise and H be a horizontal shear with factor $k = 3$. Compute the matrix for the composition: shear first, then rotate.
3. A transformation T is composed of a scaling by 2 in both directions, followed by a reflection across the line $y = x$. Find the matrix M and determine if the order of these two specific operations matters.

Solutions

1. Find the standard matrix C for the composition of a reflection across the y -axis followed by a scaling of the x -axis by a factor of 5.

- The reflection matrix is $A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$.
- The scaling matrix is $B = \begin{bmatrix} 5 & 0 \\ 0 & 1 \end{bmatrix}$.
- Since reflection is first, the product is $C = BA$.
- Compute $C = \begin{bmatrix} 5 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 0 \\ 0 & 1 \end{bmatrix}$.

Answer: $\begin{bmatrix} -5 & 0 \\ 0 & 1 \end{bmatrix}$

2. Let R be a rotation by 90° counter-clockwise and H be a horizontal shear with factor $k = 3$. Compute the matrix for the composition: shear first, then rotate.

- The shear matrix is $H = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$.
- The rotation matrix is $R = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$.
- The composition is $C = RH$.
- Compute $C = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 3 \end{bmatrix}$.

Answer: $\begin{bmatrix} 0 & -1 \\ 1 & 3 \end{bmatrix}$

3. A transformation T is composed of a scaling by 2 in both directions, followed by a reflection across the line $y = x$. Find the matrix M and determine if the order of these two specific operations matters.

- Scaling matrix $S = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$.
- Reflection across $y = x$ matrix $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.
- Composition $M = AS = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$.
- Check reverse order: $SA = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$.
- In this specific case, the order does not matter because S is a scalar multiple of the identity matrix.

Answer: $\begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$, \text{ order does not matter}