

Rotations, reflections, shears, scalings — the 2D gallery

Lesson 73 · Module 5 — Linear Transformations · Linear Algebra · dailymaths.uk/linalg

Practice problems

1. Find the standard matrix for a transformation that first reflects the plane across the y -axis and then scales the x -axis by a factor of 5.
2. Determine the matrix for a rotation of 180° counter-clockwise and describe its effect on any vector $\mathbf{v}$.
3. A transformation T is defined by the matrix $A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$. Identify this transformation from the gallery and find the image of the vector $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$.

Solutions

1. Find the standard matrix for a transformation that first reflects the plane across the y -axis and then scales the x -axis by a factor of 5.

- The reflection across the y -axis is $Ref_y = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$.
- The scaling of the x -axis by 5 is $S = \begin{bmatrix} 5 & 0 \\ 0 & 1 \end{bmatrix}$.
- The combined transformation is $A = S \cdot Ref_y = \begin{bmatrix} 5 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 0 \\ 0 & 1 \end{bmatrix}$.

Answer: $\begin{bmatrix} -5 & 0 \\ 0 & 1 \end{bmatrix}$

2. Determine the matrix for a rotation of 180° counter-clockwise and describe its effect on any vector $\mathbf{v}$.

- Using R_θ with $\theta = 180^\circ$, we have $\cos(180^\circ) = -1$ and $\sin(180^\circ) = 0$.
- The matrix is $R_{180} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$.
- Applying this to $\begin{bmatrix} x \\ y \end{bmatrix}$ gives $\begin{bmatrix} -x \\ -y \end{bmatrix}$, which is the negation of the original vector.

Answer: $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

3. A transformation T is defined by the matrix $A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$. Identify this transformation from the gallery and find the image of the vector $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$.

- The matrix has ones on the diagonal and a non-zero entry in the bottom-left. This is a vertical shear H_y with factor $k = 2$.
- The image is $A\mathbf{x} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix}$.
- The new x is $1(3) + 0(4) = 3$.
- The new y is $2(3) + 1(4) = 6 + 4 = 10$.

Answer: $\begin{bmatrix} 3 \\ 10 \end{bmatrix}$