

Every linear map is a matrix (and how to find it)

Lesson 72 · Module 5 — Linear Transformations · Linear Algebra · dailymaths.uk/linalg

Practice problems

1. Find the standard matrix A for the linear transformation $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ that scales the x -axis by 2 and the y -axis by 3.
2. A linear transformation $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is defined by the formula $T(x, y) = (x + 2y, 3x - 4y)$. Find its standard matrix A .
3. Find the standard matrix A for a transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ that maps the basis vector $\mathbf{e}_1$ to $[1, 1]^T$, $\mathbf{e}_2$ to $[0, 1]^T$, and $\mathbf{e}_3$ to $[-1, 0]^T$.

Solutions

1. Find the standard matrix A for the linear transformation $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ that scales the x -axis by 2 and the y -axis by 3.

- Find the image of the first basis vector $\mathbf{i} = [1, 0]^T$. Scaling x by 2 gives $T(\mathbf{i}) = [2, 0]^T$.
- Find the image of the second basis vector $\mathbf{j} = [0, 1]^T$. Scaling y by 3 gives $T(\mathbf{j}) = [0, 3]^T$.
- Place these images as the columns of the matrix A .

Answer: $A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$

2. A linear transformation $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is defined by the formula $T(x, y) = (x + 2y, 3x - 4y)$. Find its standard matrix A .

- Evaluate $T(1, 0)$ to find the first column: $T(1, 0) = (1 + 2(0), 3(1) - 4(0)) = (1, 3)$.
- Evaluate $T(0, 1)$ to find the second column: $T(0, 1) = (0 + 2(1), 3(0) - 4(1)) = (2, -4)$.
- Assemble the columns into matrix A .

Answer: $A = \begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix}$

3. Find the standard matrix A for a transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ that maps the basis vector $\mathbf{e}_1$ to $[1, 1]^T$, $\mathbf{e}_2$ to $[0, 1]^T$, and $\mathbf{e}_3$ to $[-1, 0]^T$.

- The images of the basis vectors are given directly as the columns of the matrix.
- Column 1 is $[1, 1]^T$.
- Column 2 is $[0, 1]^T$.
- Column 3 is $[-1, 0]^T$.
- Combine these into a 2×3 matrix.

Answer: $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 1 & 0 \end{bmatrix}$