

Coordinates relative to a basis

Lesson 66 · Module 4 — Vector Spaces · Linear Algebra · dailymaths.uk/linalg

Practice problems

1. Find the coordinates of the vector $v = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$ relative to the basis $B = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \end{bmatrix} \right\}$.
2. Given the basis $B = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \right\}$, find the coordinates of $v = \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}$.
3. If a vector v has coordinates $[v]_B = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$ relative to the basis $B = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$, what are its coordinates relative to the standard basis?

Solutions

1. Find the coordinates of the vector $v = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$ relative to the basis $B = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \end{bmatrix} \right\}$.

- Set up the equation $\begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$.
- From the second row, $1c_1 + 0c_2 = 2$, so $c_1 = 2$.
- Substitute $c_1 = 2$ into the first row: $2 + 2c_2 = 5$.
- Subtract 2 from both sides: $2c_2 = 3$.
- Divide by 2: $c_2 = 1.5$.

Answer: $[v]_B = \begin{bmatrix} 2 \\ 1.5 \end{bmatrix}$

2. Given the basis $B = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \right\}$, find the coordinates of $v = \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}$.

- Set up the system $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}$.
- Subtract row 1 from row 3: $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$.
- Subtract row 2 from row 3: $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}$.
- Solve for c_3 : $-2c_3 = -1 \implies c_3 = 0.5$.
- Solve for c_2 : $c_2 + 0.5 = 2 \implies c_2 = 1.5$.
- Solve for c_1 : $c_1 + 0.5 = 2 \implies c_1 = 1.5$.

Answer: $[v]_B = \begin{bmatrix} 1.5 \\ 1.5 \\ 0.5 \end{bmatrix}$

3. If a vector v has coordinates $[v]_B = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$ relative to the basis $B = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$, what are its coordinates relative to the standard basis?

- The coordinates relative to the standard basis are simply the vector v itself.
- Use the formula $v = P[v]_B$.
- Calculate $v = 3 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + (-2) \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.
- Compute the components: $x = 3(1) + (-2)(0) = 3$ and $y = 3(2) + (-2)(1) = 6 - 2 = 4$.
- The vector is $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$.

Answer: $[v]_{std} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$