

The Rank-Nullity Theorem — the great accounting identity

Lesson 64 · Module 4 — Vector Spaces · Linear Algebra · dailymaths.uk/linalg

Practice problems

1. A matrix A has 6 columns and a rank of 4. What is the nullity of A ?
2. A 4×7 matrix A has a null space spanned by the vectors $[1, 0, 0, 0, 0, 0, 0]^T$, $[0, 1, 0, 0, 0, 0, 0]^T$, and $[0, 0, 1, 0, 0, 0, 0]^T$. Find the rank of A .
3. Let A be a 3×3 matrix. If the system $Ax = 0$ has only the trivial solution, what is the rank of A ?

Solutions

1. A matrix A has 6 columns and a rank of 4. What is the nullity of A ?

- Identify the number of columns $n = 6$.
- Identify the rank $\text{rank}(A) = 4$.
- Apply the Rank-Nullity Theorem: $\text{rank}(A) + \text{nullity}(A) = n$.
- Substitute the values: $4 + \text{nullity}(A) = 6$.
- Solve for nullity: $\text{nullity}(A) = 6 - 4 = 2$.

Answer: 2

2. A 4×7 matrix A has a null space spanned by the vectors $[1, 0, 0, 0, 0, 0, 0]^T$, $[0, 1, 0, 0, 0, 0, 0]^T$, and $[0, 0, 1, 0, 0, 0, 0]^T$. Find the rank of A .

- The null space is spanned by 3 linearly independent vectors, so $\text{nullity}(A) = 3$.
- The number of columns n is 7.
- Apply the Rank-Nullity Theorem: $\text{rank}(A) + 3 = 7$.
- Solve for rank: $\text{rank}(A) = 7 - 3 = 4$.

Answer: 4

3. Let A be a 3×3 matrix. If the system $Ax = 0$ has only the trivial solution, what is the rank of A ?

- If $Ax = 0$ has only the trivial solution, the null space contains only the zero vector.
- The dimension of the zero subspace is 0, so $\text{nullity}(A) = 0$.
- The number of columns n is 3.
- Apply the Rank-Nullity Theorem: $\text{rank}(A) + 0 = 3$.
- Therefore, $\text{rank}(A) = 3$.

Answer: 3