

The row space; rank of a matrix

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Practice problems

1. Find the rank and a basis for the row space of the matrix $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 3 & 6 & 3 \end{bmatrix}$.
2. Given a 4×6 matrix B with 3 pivots, what is the dimension of the row space and the dimension of the column space?
3. Find a basis for the row space of $C = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 2 \end{bmatrix}$.

Solutions

1. Find the rank and a basis for the row space of the matrix $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 3 & 6 & 3 \end{bmatrix}$.

- Perform row operations: $R_2 \leftarrow R_2 - 2R_1$ and $R_3 \leftarrow R_3 - 3R_1$.
- The resulting matrix is $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.
- There is only one non-zero row, so the rank is 1.
- The basis for the row space is the set containing the single non-zero row.

Answer: $\text{rank}(A) = 1, \text{Basis}(\text{Row } A) = \{ (1, 2, 1) \}$

2. Given a 4×6 matrix B with 3 pivots, what is the dimension of the row space and the dimension of the column space?

- The number of pivots is given as 3.
- The dimension of the column space is equal to the number of pivots.
- The dimension of the row space is also equal to the number of pivots.
- Therefore, both dimensions are 3.

Answer: $\text{dim}(\text{Row } B) = 3, \text{dim}(\text{Col } B) = 3$

3. Find a basis for the row space of $C = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 2 \end{bmatrix}$.

- Perform row operations: $R_3 \leftarrow R_3 - R_1$.
- The matrix becomes $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$.
- Perform $R_3 \leftarrow R_3 - R_2$.
- The RREF is $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$.
- The non-zero rows are $(1, 0, 1)$ and $(0, 1, 1)$.

Answer: $\text{Basis}(\text{Row } C) = \{ (1, 0, 1), (0, 1, 1) \}$