

# Dimension — and why it's well-defined

Lesson 61 · Module 4 — Vector Spaces · Linear Algebra · [dailymaths.uk/linalg](http://dailymaths.uk/linalg)

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## Practice problems

1. Find the dimension of the subspace  $V$  of  $\mathbb{R}^3$  defined by the equation  $2x - y + 3z = 0$ .
2. Given a  $4 \times 7$  matrix  $A$  with a rank of 3, what is the dimension of the null space of  $A$ ?
3. Determine the dimension of the subspace of  $\mathbb{R}^3$  spanned by the vectors  $v_1 = (1, 0, 0)$ ,  $v_2 = (0, 1, 0)$ , and  $v_3 = (1, 1, 0)$ .

## Solutions

1. Find the dimension of the subspace  $V$  of  $\mathbb{R}^3$  defined by the equation  $2x - y + 3z = 0$ .

- Solve for one variable in terms of the others:  $y = 2x + 3z$ .
- Write a general vector in  $V$ :  $(x, 2x + 3z, z)$ .
- Decompose into linear combinations:  $x(1, 2, 0) + z(0, 3, 1)$ .
- The vectors  $(1, 2, 0)$  and  $(0, 3, 1)$  are linearly independent and span  $V$ .
- The basis has 2 vectors.

**Answer:** 2

2. Given a  $4 \times 7$  matrix  $A$  with a rank of 3, what is the dimension of the null space of  $A$ ?

- Use the Rank-Nullity Theorem:  $\text{rank}(A) + \dim(\text{Null } A) = n$ , where  $n$  is the number of columns.
- Substitute the known values:  $3 + \dim(\text{Null } A) = 7$ .
- Solve for the dimension of the null space:  $7 - 3 = 4$ .

**Answer:** 4

3. Determine the dimension of the subspace of  $\mathbb{R}^3$  spanned by the vectors  $v_1 = (1, 0, 0)$ ,  $v_2 = (0, 1, 0)$ , and  $v_3 = (1, 1, 0)$ .

- Check for linear independence:  $v_3 = v_1 + v_2$ .
- Since  $v_3$  is a linear combination of  $v_1$  and  $v_2$ , the set is linearly dependent.
- The spanning set can be reduced to the linearly independent set  $\{v_1, v_2\}$ .
- This set forms a basis for the subspace.
- The number of vectors in the basis is 2.

**Answer:** 2