

Basis: minimal spanning sets

Lesson 60 · Module 4 — Vector Spaces · Linear Algebra · dailymaths.uk/linalg

Practice problems

1. Determine if the set $S = \left\{ \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \end{bmatrix} \right\}$ is a basis for $\mathbb{R}^2$.
2. Find the coordinates of the vector $w = \begin{bmatrix} 4 \\ 10 \end{bmatrix}$ relative to the basis $B = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\}$.
3. Does the set $S = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \right\}$ form a basis for $\mathbb{R}^3$?

Solutions

1. Determine if the set $S = \left\{ \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \end{bmatrix} \right\}$ is a basis for $\mathbb{R}^2$.

- Form a matrix A with the vectors as columns: $A = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$.
- Calculate the determinant: $\det(A) = (2)(3) - (1)(1) = 6 - 1 = 5$.
- Since the determinant is non-zero, the vectors are linearly independent.
- In $\mathbb{R}^2$, any two linearly independent vectors span the space.
- Therefore, the set is a basis.

Answer: Yes, it is a basis.

2. Find the coordinates of the vector $w = \begin{bmatrix} 4 \\ 10 \end{bmatrix}$ relative to the basis $B = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\}$.

- Set up the equation $c_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ 10 \end{bmatrix}$.
- From the second component: $2c_1 + 0c_2 = 10$, which implies $c_1 = 5$.
- From the first component: $1c_1 + 1c_2 = 4$.
- Substitute $c_1 = 5$: $5 + c_2 = 4$, which implies $c_2 = -1$.
- The coordinates are $c_1 = 5$ and $c_2 = -1$.

Answer: $\begin{bmatrix} 5 \\ -1 \end{bmatrix}$

3. Does the set $S = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \right\}$ form a basis for $\mathbb{R}^3$?

- Check for linear independence by calculating the determinant of the matrix $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$.
- The determinant of an upper triangular matrix is the product of the diagonal: $1 \cdot 1 \cdot 0 = 0$.
- Since the determinant is zero, the vectors are linearly dependent.
- A basis must be linearly independent.
- Therefore, the set is not a basis.

Answer: No, it is not a basis.