

Linear independence II — testing with elimination

Lesson 59 · Module 4 — Vector Spaces · Linear Algebra · dailymaths.uk/linalg

Practice problems

1. Determine if the following vectors in $\mathbb{R}^3$ are linearly independent: $v_1 = [1, 0, 1]^T$, $v_2 = [1, 1, 0]^T$, and $v_3 = [0, 1, 1]^T$.
2. Check the linear independence of $v_1 = [1, 2]^T$ and $v_2 = [3, 6]^T$.
3. Given $v_1 = [1, 0, 0]^T$, $v_2 = [1, 1, 0]^T$, and $v_3 = [1, 1, 1]^T$, are they linearly independent? Explain using pivots.

Solutions

1. Determine if the following vectors in $\mathbb{R}^3$ are linearly independent: $v_1 = [1, 0, 1]^T$, $v_2 = [1, 1, 0]^T$, and $v_3 = [0, 1, 1]^T$.

- Form the matrix $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$.
- Perform $R_3 \leftarrow R_3 - R_1$ to get $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & -1 & 1 \end{bmatrix}$.
- Perform $R_3 \leftarrow R_3 + R_2$ to get $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix}$.
- There are 3 pivots for 3 vectors.

Answer: Linearly Independent

2. Check the linear independence of $v_1 = [1, 2]^T$ and $v_2 = [3, 6]^T$.

- Form the matrix $A = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}$.
- Perform $R_2 \leftarrow R_2 - 2R_1$ to get $\begin{bmatrix} 1 & 3 \\ 0 & 0 \end{bmatrix}$.
- There is only 1 pivot, but there are 2 vectors.

Answer: Linearly Dependent

3. Given $v_1 = [1, 0, 0]^T$, $v_2 = [1, 1, 0]^T$, and $v_3 = [1, 1, 1]^T$, are they linearly independent? Explain using pivots.

- Form the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$.
- The matrix is already in row echelon form.
- There are pivots in every column (the diagonal entries are all 1).
- Since the number of pivots equals the number of vectors, they are independent.

Answer: Linearly Independent