

The null space of a matrix

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Practice problems

1. Find the null space of the matrix $A = \begin{pmatrix} 1 & -3 \\ 2 & -6 \end{pmatrix}$.
2. Find a basis for the null space of $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix}$.
3. Determine the null space of $A = \begin{pmatrix} 1 & 2 & 0 & 1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$.

Solutions

1. Find the null space of the matrix $A = \begin{pmatrix} 1 & -3 \\ 2 & -6 \end{pmatrix}$.

- Perform row reduction: $R_2 \leftarrow R_2 - 2R_1$ gives $\begin{pmatrix} 1 & -3 \\ 0 & 0 \end{pmatrix}$.
- Identify variables: x_1 is a pivot variable, x_2 is a free variable.
- Solve for x_1 : $x_1 - 3x_2 = 0 \implies x_1 = 3x_2$.
- Write in vector form: $x = \begin{pmatrix} 3x_2 \\ x_2 \end{pmatrix} = x_2 \begin{pmatrix} 3 \\ 1 \end{pmatrix}$.

Answer: $\text{Null}(A) = \text{span} \left\{ \begin{pmatrix} 3 \\ 1 \end{pmatrix} \right\}$

2. Find a basis for the null space of $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix}$.

- The matrix is already in RREF.
- Pivot variables are x_1 and x_2 ; the free variable is x_3 .
- From row 1: $x_1 + 2x_3 = 0 \implies x_1 = -2x_3$.
- From row 2: $x_2 - x_3 = 0 \implies x_2 = x_3$.
- The general solution is $x = x_3 \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$.

Answer: $\left\{ \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} \right\}$

3. Determine the null space of $A = \begin{pmatrix} 1 & 2 & 0 & 1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$.

- The matrix is in RREF. Pivots are in columns 1 and 3.
- Free variables are x_2 and x_4 .
- Row 1: $x_1 + 2x_2 + x_4 = 0 \implies x_1 = -2x_2 - x_4$.
- Row 2: $x_3 + 3x_4 = 0 \implies x_3 = -3x_4$.
- Vector form: $x = x_2 \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \end{pmatrix} + x_4 \begin{pmatrix} -1 \\ 0 \\ -3 \\ 1 \end{pmatrix}$.

Answer: $\text{Null}(A) = \text{span} \left\{ \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ -3 \\ 1 \end{pmatrix} \right\}$