

# Subspaces: definition and recognition

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## Practice problems

1. Determine if the set  $S = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} : 2x - y = 0 \right\}$  is a subspace of  $\mathbb{R}^3$ .
2. Is the set  $S = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : x^2 = y \right\}$  a subspace of  $\mathbb{R}^2$ ?
3. Let  $S$  be the set of all vectors in  $\mathbb{R}^3$  that are perpendicular to the vector  $n = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ . Is  $S$  a subspace?

## Solutions

1. Determine if the set  $S = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} : 2x - y = 0 \right\}$  is a subspace of  $\mathbb{R}^3$ .
- Check zero vector:  $2(0) - 0 = 0$ , so  $0 \in S$ .
  - Check addition: If  $2u_1 - u_2 = 0$  and  $2v_1 - v_2 = 0$ , then  $2(u_1 + v_1) - (u_2 + v_2) = (2u_1 - u_2) + (2v_1 - v_2) = 0 + 0 = 0$ .
  - Check scalar multiplication: If  $2x - y = 0$ , then  $2(cx) - (cy) = c(2x - y) = c(0) = 0$ .

**Answer:** Yes, it is a subspace.

2. Is the set  $S = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : x^2 = y \right\}$  a subspace of  $\mathbb{R}^2$ ?
- Check zero vector:  $0^2 = 0$ , so  $0 \in S$ .
  - Check scalar multiplication: Let  $v = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ . This is in  $S$  because  $1^2 = 1$ .
  - Multiply by  $c = 2$ :  $2v = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$ . Check if  $2^2 = 2$ . Since  $4 \neq 2$ , the vector  $2v$  is not in  $S$ .

**Answer:** No, it is not a subspace.

3. Let  $S$  be the set of all vectors in  $\mathbb{R}^3$  that are perpendicular to the vector  $n = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ . Is  $S$  a subspace?

- A vector  $v$  is in  $S$  if  $n \cdot v = 0$ .
- Zero vector:  $n \cdot 0 = 0$ , so  $0 \in S$ .
- Addition: If  $n \cdot u = 0$  and  $n \cdot v = 0$ , then  $n \cdot (u + v) = n \cdot u + n \cdot v = 0 + 0 = 0$ .
- Scalar multiplication: If  $n \cdot v = 0$ , then  $n \cdot (cv) = c(n \cdot v) = c(0) = 0$ .

**Answer:** Yes, it is a subspace.