

What is a vector space? Abstracting the rules

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Practice problems

1. Determine if the set $V = \{[x, y, z] \mid x + y + z = 0\}$ is a vector space.
2. Is the set of all 2×2 matrices with a determinant of 0 a vector space?
3. Consider the set of all functions $f(x)$ such that $f(0) = 1$. Is this a vector space?

Solutions

1. Determine if the set $V = \{[x, y, z] \mid x + y + z = 0\}$ is a vector space.

- Check closure under addition: if $u = [x_1, y_1, z_1]$ and $v = [x_2, y_2, z_2]$ are in V , then $x_1 + y_1 + z_1 = 0$ and $x_2 + y_2 + z_2 = 0$. Their sum is $[x_1 + x_2, y_1 + y_2, z_1 + z_2]$. The sum of these components is $(x_1 + y_1 + z_1) + (x_2 + y_2 + z_2) = 0 + 0 = 0$. So it is closed under addition.
- Check closure under scalar multiplication: $cu = [cx_1, cy_1, cz_1]$. The sum of components is $c(x_1 + y_1 + z_1) = c(0) = 0$. So it is closed under scalar multiplication.
- Check for zero vector: $[0, 0, 0]$ satisfies $0 + 0 + 0 = 0$, so it is in V .
- Since it is a subset of $\mathbb{R}^3$ and satisfies these properties, it is a vector space.

Answer: Yes, it is a vector space.

2. Is the set of all 2×2 matrices with a determinant of 0 a vector space?

- Check closure under addition. Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$.
- Both have $\det(A) = 0$ and $\det(B) = 0$, so they are in the set.
- The sum is $A + B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.
- The determinant of the sum is $\det(A + B) = 1 \cdot 1 - 0 \cdot 0 = 1$.
- Since the sum is not in the set, it is not closed under addition.

Answer: No, it is not a vector space.

3. Consider the set of all functions $f(x)$ such that $f(0) = 1$. Is this a vector space?

- Check for the zero vector. The zero vector in the space of functions is the function $z(x) = 0$ for all x .
- For this function, $z(0) = 0$.
- However, the set requires $f(0) = 1$.
- Since the zero function is not in the set, it cannot be a vector space.

Answer: No, it is not a vector space.