

Module 3 review + self-test

Lesson 53 · Module 3 — Determinants · Linear Algebra · dailymaths.uk/linalg

Practice problems

1. Compute the determinant of the matrix $A = \begin{pmatrix} 2 & 0 & 0 \\ 1 & 3 & 0 \\ 4 & -1 & 2 \end{pmatrix}$.
2. If A is a 3×3 matrix with $\det(A) = -2$, find $\det(5A^T)$.
3. Determine if the matrix $M = \begin{pmatrix} 1 & 2 \\ 3 & 6 \end{pmatrix}$ is invertible and explain why using the determinant.

Solutions

1. Compute the determinant of the matrix $A = \begin{pmatrix} 2 & 0 & 0 \\ 1 & 3 & 0 \\ 4 & -1 & 2 \end{pmatrix}$.

- Notice that the matrix is lower triangular.
- The determinant of a triangular matrix is the product of its diagonal entries.
- Calculate $2 \times 3 \times 2 = 12$.

Answer: 12

2. If A is a 3×3 matrix with $\det(A) = -2$, find $\det(5A^T)$.

- The transpose does not change the determinant: $\det(A^T) = \det(A) = -2$.
- For a 3×3 matrix, $\det(kA) = k^3 \det(A)$.
- Substitute $k = 5$ and $\det(A^T) = -2$: $5^3 \times (-2) = 125 \times (-2) = -250$.

Answer: -250

3. Determine if the matrix $M = \begin{pmatrix} 1 & 2 \\ 3 & 6 \end{pmatrix}$ is invertible and explain why using the determinant.

- Compute the determinant: $\det(M) = (1 \times 6) - (2 \times 3)$.
- Calculate $6 - 6 = 0$.
- Since the determinant is zero, the matrix is singular and therefore not invertible.

Answer: Not invertible because $\det(M) = 0$