

Determinant clinic — mixed practice

Lesson 52 · Module 3 — Determinants · Linear Algebra · dailymaths.uk/linalg

Practice problems

1. Compute the determinant of $A = \begin{pmatrix} 2 & 0 & 1 \\ 5 & 0 & 3 \\ 1 & 4 & 2 \end{pmatrix}$ using the most efficient method.
2. Find the determinant of $B = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}^3$.
3. Determine if the matrix $C = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 0 & 1 & 3 \end{pmatrix}$ is invertible by calculating its determinant.

Solutions

1. Compute the determinant of $A = \begin{pmatrix} 2 & 0 & 1 \\ 5 & 0 & 3 \\ 1 & 4 & 2 \end{pmatrix}$ using the most efficient method.
- Notice that the second column has two zeros.
 - Expand along the second column. The only non-zero entry is 4 at position $(3, 2)$.
 - The sign for position $(3, 2)$ is negative.
 - The determinant is $-4 \times \det \begin{pmatrix} 2 & 1 \\ 5 & 3 \end{pmatrix}$.
 - Calculate the 2×2 determinant: $(2 \times 3) - (1 \times 5) = 6 - 5 = 1$.
 - The final result is $-4 \times 1 = -4$.

Answer: -4

2. Find the determinant of $B = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}^3$.
- First, find the determinant of the base matrix B .
 - $\det(B) = (1 \times 4) - (2 \times 3) = 4 - 6 = -2$.
 - Use the property $\det(B^k) = (\det(B))^k$.
 - The determinant is $(-2)^3$.
 - Calculate $(-2) \times (-2) \times (-2) = -8$.

Answer: -8

3. Determine if the matrix $C = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 0 & 1 & 3 \end{pmatrix}$ is invertible by calculating its determinant.
- Observe the rows of the matrix.
 - Row 2 is exactly 2 times Row 1.
 - Since the rows are linearly dependent, the determinant must be 0.
 - Because $\det(C) = 0$, the matrix is singular.
 - Therefore, the matrix is not invertible.

Answer: Not invertible ($\det(C) = 0$)