

The determinant test for invertibility — tying it together

Lesson 51 · Module 3 — Determinants · Linear Algebra · dailymaths.uk/linalg

Practice problems

1. Determine if the matrix $A = \begin{pmatrix} 4 & 6 \\ 2 & 3 \end{pmatrix}$ is invertible. Explain your reasoning using the determinant.
2. Given a 3×3 matrix B where $\det(B) = -5$, is the system $Bx = 0$ guaranteed to have only the trivial solution $x = 0$? Why?
3. Consider the matrix $C = \begin{pmatrix} k & 2 \\ 3 & 6 \end{pmatrix}$. For what value of k is this matrix singular?

Solutions

1. Determine if the matrix $A = \begin{pmatrix} 4 & 6 \\ 2 & 3 \end{pmatrix}$ is invertible. Explain your reasoning using the determinant.

- Compute the determinant: $\det(A) = (4)(3) - (6)(2)$
- Calculate the values: $12 - 12 = 0$
- Since the determinant is zero, the matrix is singular.

Answer: Not invertible (singular) because $\det(A) = 0$.

2. Given a 3×3 matrix B where $\det(B) = -5$, is the system $Bx = 0$ guaranteed to have only the trivial solution $x = 0$? Why?

- The determinant $\det(B) = -5$ is non zero.
- A non zero determinant implies the matrix B is invertible.
- If B is invertible, we can multiply $Bx = 0$ by B^{-1} on the left.
- This gives $x = B^{-1}0$, which simplifies to $x = 0$.

Answer: Yes, because $\det(B) \neq 0$ implies B is invertible, ensuring a unique solution.

3. Consider the matrix $C = \begin{pmatrix} k & 2 \\ 3 & 6 \end{pmatrix}$. For what value of k is this matrix singular?

- A matrix is singular if its determinant is zero.
- Set up the determinant equation: $(k)(6) - (2)(3) = 0$
- Simplify the equation: $6k - 6 = 0$
- Solve for k : $6k = 6$, so $k = 1$.

Answer: $k = 1$