

Cramer's rule (and why you rarely use it)

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Practice problems

1. Use Cramer's rule to find the value of x for the system: $3x + 2y = 7$ and $4x - y = 2$.
2. Solve the system $x + y = 5$ and $2x + 3y = 13$ completely using Cramer's rule.
3. Consider the system $ax + by = e$ and $cx + dy = f$. Use Cramer's rule to write the general formula for x in terms of the coefficients.

Solutions

1. Use Cramer's rule to find the value of x for the system: $3x + 2y = 7$ and $4x - y = 2$.

- Identify $A = \begin{bmatrix} 3 & 2 \\ 4 & -1 \end{bmatrix}$ and $b = \begin{bmatrix} 7 \\ 2 \end{bmatrix}$.
- Compute $\det(A) = (3)(-1) - (2)(4) = -3 - 8 = -11$.
- Construct $A_1 = \begin{bmatrix} 7 & 2 \\ 2 & -1 \end{bmatrix}$.
- Compute $\det(A_1) = (7)(-1) - (2)(2) = -7 - 4 = -11$.
- Calculate $x = \frac{-11}{-11} = 1$.

Answer: $x = 1$

2. Solve the system $x + y = 5$ and $2x + 3y = 13$ completely using Cramer's rule.

- $\det(A) = (1)(3) - (1)(2) = 1$.
- $\det(A_1) = (5)(3) - (1)(13) = 15 - 13 = 2$.
- $\det(A_2) = (1)(13) - (5)(2) = 13 - 10 = 3$.
- Calculate $x = 2/1 = 2$ and $y = 3/1 = 3$.

Answer: $x = 2, y = 3$

3. Consider the system $ax + by = e$ and $cx + dy = f$. Use Cramer's rule to write the general formula for x in terms of the coefficients.

- The coefficient matrix is $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$.
- The determinant is $\det(A) = ad - bc$.
- The modified matrix for x is $A_1 = \begin{bmatrix} e & b \\ f & d \end{bmatrix}$.
- The determinant is $\det(A_1) = ed - bf$.
- The formula is $x = \frac{ed - bf}{ad - bc}$.

Answer: $x = \frac{ed - bf}{ad - bc}$