

$\det(\mathbf{AB}) = \det(\mathbf{A})\det(\mathbf{B})$ and consequences

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Practice problems

1. Given $A = \begin{pmatrix} 2 & 1 \\ 0 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 \\ 4 & 2 \end{pmatrix}$, compute $\det(AB)$ using the multiplicativity property.
2. A 3×3 matrix M has $\det(M) = -5$. What is the determinant of M^3 ?
3. If $\det(A) = 2$ and $\det(B) = 0.5$, find $\det(A^2B^{-1})$.

Solutions

1. Given $A = \begin{pmatrix} 2 & 1 \\ 0 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 \\ 4 & 2 \end{pmatrix}$, compute $\det(AB)$ using the multiplicativity property.
- Compute $\det(A) = (2)(3) - (1)(0) = 6$.
 - Compute $\det(B) = (1)(2) - (0)(4) = 2$.
 - Apply the property $\det(AB) = \det(A) \det(B) = 6 \times 2 = 12$.

Answer: 12

2. A 3×3 matrix M has $\det(M) = -5$. What is the determinant of M^3 ?
- Use the power property $\det(M^k) = (\det(M))^k$.
 - Substitute $k = 3$ and $\det(M) = -5$.
 - Calculate $(-5)^3 = (-5) \times (-5) \times (-5) = -125$.

Answer: -125

3. If $\det(A) = 2$ and $\det(B) = 0.5$, find $\det(A^2B^{-1})$.
- Use the property $\det(A^2B^{-1}) = \det(A)^2 \times \det(B^{-1})$.
 - Calculate $\det(A)^2 = 2^2 = 4$.
 - Calculate $\det(B^{-1}) = 1/\det(B) = 1/0.5 = 2$.
 - Multiply the results: $4 \times 2 = 8$.

Answer: 8