

Determinants via row reduction — the practical method

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Practice problems

1. Compute the determinant of the matrix $A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$ using row reduction.
2. Find the determinant of $B = \begin{bmatrix} 0 & 2 \\ 3 & 4 \end{bmatrix}$ using the row reduction method.
3. Determine if the matrix $C = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 0 & 1 \end{bmatrix}$ is singular by attempting to find its determinant via row reduction.

Solutions

1. Compute the determinant of the matrix $A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$ using row reduction.

- Step 1: Replace R_2 with $R_2 - 0.5R_1$. The matrix becomes $\begin{bmatrix} 2 & 1 & 0 \\ 0 & 1.5 & 1 \\ 0 & 1 & 2 \end{bmatrix}$. Determinant is unchanged.
- Step 2: Replace R_3 with $R_3 - (1/1.5)R_2$, which is $R_3 - (2/3)R_2$. The matrix becomes $\begin{bmatrix} 2 & 1 & 0 \\ 0 & 1.5 & 1 \\ 0 & 0 & 2 - 2/3 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1.5 & 1 \\ 0 & 0 & 4/3 \end{bmatrix}$. Determinant is unchanged.
- Step 3: Multiply the diagonal elements: $2 \cdot 1.5 \cdot (4/3) = 2 \cdot (3/2) \cdot (4/3) = 4$.

Answer: 4

2. Find the determinant of $B = \begin{bmatrix} 0 & 2 \\ 3 & 4 \end{bmatrix}$ using the row reduction method.

- Step 1: Swap R_1 and R_2 to get a non-zero pivot. The matrix becomes $\begin{bmatrix} 3 & 4 \\ 0 & 2 \end{bmatrix}$. This introduces a factor of -1 .
- Step 2: The matrix is now upper triangular. The product of the pivots is $3 \cdot 2 = 6$.
- Step 3: Apply the swap factor: $(-1) \cdot 6 = -6$.

Answer: -6

3. Determine if the matrix $C = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 0 & 1 \end{bmatrix}$ is singular by attempting to find its determinant via row reduction.

- Step 1: Replace R_2 with $R_2 - 2R_1$. The matrix becomes $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}$.
- Step 2: We observe that the second row has become all zeros.
- Step 3: Any triangular matrix with a zero on the diagonal has a determinant of 0. Therefore, $\det(C) = 0$ and the matrix is singular.

Answer: 0 (Singular)