Cofactor Expansion

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Practice problems

1. Compute the determinant of the matrix $A = \begin{bmatrix} 2 & 0 & 1 \\ 3 & 0 & 4 \\ 1 & 5 & 2 \end{bmatrix}$ using cofactor expansion.
2. Find the determinant of $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ using cofactor expansion along the first row to verify it matches the 2x2 formula.
3. Calculate the determinant of $C = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & 0 \\ 0 & 4 & 5 \end{bmatrix}$ by expanding along the first column.

Solutions

1. Compute the determinant of the matrix $A = \begin{bmatrix} 2 & 0 & 1 \\ 3 & 0 & 4 \\ 1 & 5 & 2 \end{bmatrix}$ using cofactor expansion.

   - Choose the second column for expansion because it has two zeros.
   - The only non-zero element is $a_{32} = 5$.
   - The cofactor sign for row 3, column 2 is $(-1)^{3+2} = -1$.
   - The minor M_{32} is the determinant of $\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$, which is $(2 \cdot 4) - (1 \cdot 3) = 8 - 3 = 5$.
   - The determinant is $a_{32} \cdot C_{32} = 5 \cdot (-1 \cdot 5) = -25$.

   Answer: -25

2. Find the determinant of $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ using cofactor expansion along the first row to verify it matches the 2x2 formula.

   - Expand along row 1: elements are 1 and 2.
   - First term: $1 \cdot C_{11} = 1 \cdot (-1)^{1+1} \cdot \det[4] = 1 \cdot 1 \cdot 4 = 4$.
   - Second term: $2 \cdot C_{12} = 2 \cdot (-1)^{1+2} \cdot \det[3] = 2 \cdot (-1) \cdot 3 = -6$.
   - Sum: $4 - 6 = -2$.

   Answer: -2

3. Calculate the determinant of $C = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & 0 \\ 0 & 4 & 5 \end{bmatrix}$ by expanding along the first column.

   - Expand along column 1: elements are 1, 2, 0.
   - Term 1: $1 \cdot C_{11} = 1 \cdot (-1)^{1+1} \cdot \det \begin{bmatrix} 1 & 0 \\ 4 & 5 \end{bmatrix} = 1 \cdot 1 \cdot (5 - 0) = 5$.
   - Term 2: $2 \cdot C_{21} = 2 \cdot (-1)^{2+1} \cdot \det \begin{bmatrix} 0 & 3 \\ 4 & 5 \end{bmatrix} = 2 \cdot (-1) \cdot (0 - 12) = -2 \cdot (-12) = 24$.
   - Term 3: $0 \cdot C_{31} = 0$.
   - Sum: $5 + 24 + 0 = 29$.

   Answer: 29