

Properties of determinants (from the geometric view)

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Practice problems

1. Find the determinant of the matrix $A = \begin{bmatrix} 2 & 4 \\ 5 & 10 \end{bmatrix}$ using the properties learned today.
2. If $\det(A) = 5$, what is the determinant of the matrix B obtained by swapping two rows of A and then multiplying the first row by 3?
3. Calculate the determinant of $M = \begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}$ by using the row operation $R_2 \rightarrow R_2 - 3R_1$.

Solutions

1. Find the determinant of the matrix $A = \begin{bmatrix} 2 & 4 \\ 5 & 10 \end{bmatrix}$ using the properties learned today.

- Observe that the second row $[5, 10]$ is 2.5 times the first row $[2, 4]$.
- Since the rows are linearly dependent, the vectors are collinear.
- The area of the resulting parallelogram is zero.

Answer: 0

2. If $\det(A) = 5$, what is the determinant of the matrix B obtained by swapping two rows of A and then multiplying the first row by 3?

- Starting determinant is 5.
- Swapping two rows multiplies the determinant by -1 , giving -5 .
- Multiplying one row by 3 multiplies the determinant by 3, giving $-5 \times 3 = -15$.

Answer: -15

3. Calculate the determinant of $M = \begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}$ by using the row operation $R_2 \rightarrow R_2 - 3R_1$.

- Perform the operation: R_2 becomes $[3 - 3(1), 7 - 3(2)] = [0, 1]$.
- The new matrix is $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$.
- The row addition property states the determinant remains unchanged.
- The determinant of the resulting matrix is $1 \times 1 - 2 \times 0 = 1$.

Answer: 1