

2x2 and 3x3 determinants; orientation and sign

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Practice problems

1. Compute the determinant of the matrix $A = \begin{bmatrix} 5 & 2 \\ 3 & 4 \end{bmatrix}$.
2. Compute the determinant of the matrix $B = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 3 \\ 2 & 0 & 1 \end{bmatrix}$ using the rule of Sarrus.
3. Does the transformation represented by $C = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ preserve or reverse orientation? Explain using the determinant.

Solutions

1. Compute the determinant of the matrix $A = \begin{bmatrix} 5 & 2 \\ 3 & 4 \end{bmatrix}$.

- Identify the entries: $a = 5, b = 2, c = 3, d = 4$.
- Apply the 2x2 formula: $\det(A) = ad - bc$.
- Calculate the products: $5 \times 4 = 20$ and $2 \times 3 = 6$.
- Subtract the results: $20 - 6 = 14$.

Answer: 14

2. Compute the determinant of the matrix $B = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 3 \\ 2 & 0 & 1 \end{bmatrix}$ using the rule of Sarrus.

- Calculate downward diagonals: $(1 \times 1 \times 1) + (2 \times 3 \times 2) + (1 \times 0 \times 0) = 1 + 12 + 0 = 13$.
- Calculate upward diagonals: $(2 \times 1 \times 1) + (0 \times 3 \times 1) + (1 \times 0 \times 2) = 2 + 0 + 0 = 2$.
- Subtract the upward sum from the downward sum: $13 - 2 = 11$.

Answer: 11

3. Does the transformation represented by $C = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ preserve or reverse orientation? Explain using the determinant.

- Compute the determinant: $\det(C) = (0 \times 0) - (1 \times 1)$.
- The result is $0 - 1 = -1$.
- Since the determinant is negative, the transformation reverses the orientation of the plane.

Answer: Reverses orientation because $\det(C) = -1$.