Module 2 Review and Self-Test

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Practice problems

1. Find the inverse of $A = \begin{bmatrix} 4 & 3 \\ 3 & 2 \end{bmatrix}$ and use it to solve $Ax = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.
2. Given block matrices $M = \begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix}$ and $N = \begin{bmatrix} C & 0 \\ 0 & D \end{bmatrix}$ where $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$, $C = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$, and $D = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$, compute MN.
3. Perform an LU decomposition on $A = \begin{bmatrix} 2 & 1 \\ 8 & 7 \end{bmatrix}$ and verify that $LU = A$.

Solutions

1. Find the inverse of $A = \begin{bmatrix} 4 & 3 \\ 3 & 2 \end{bmatrix}$ and use it to solve $Ax = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

   - Compute the determinant: $\det(A) = (4)(2) - (3)(3) = 8 - 9 = -1$.
   - Find the inverse: $A^{-1} = \frac{1}{-1} \begin{bmatrix} 2 & -3 \\ -3 & 4 \end{bmatrix} = \begin{bmatrix} -2 & 3 \\ 3 & -4 \end{bmatrix}$.
   - Solve for x: $x = A^{-1}b = \begin{bmatrix} -2 & 3 \\ 3 & -4 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$.

   Answer: x = \beginbmatrix -2 \\ 3 \endbmatrix

2. Given block matrices $M = \begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix}$ and $N = \begin{bmatrix} C & 0 \\ 0 & D \end{bmatrix}$ where $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$, $C = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$, and $D = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$, compute MN.

   - Use block multiplication: $MN = \begin{bmatrix} AC & 0 \\ 0 & BD \end{bmatrix}$.
   - Compute $AC = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$.
   - Compute $BD = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$.
   - Assemble the result: $MN = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 8 & 0 \\ 0 & 0 & 0 & 8 \end{bmatrix}$.

   Answer: \beginbmatrix 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 8 & 0 \\ 0 & 0 & 0 & 8 \endbmatrix

3. Perform an LU decomposition on $A = \begin{bmatrix} 2 & 1 \\ 8 & 7 \end{bmatrix}$ and verify that $LU = A$.

   - Eliminate the 8 in the second row: $R_2 \leftarrow R_2 - 4R_1$.
   - The resulting upper triangular matrix is $U = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$.
   - The multiplier was 4, so the lower triangular matrix is $L = \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix}$.
   - Verify: $LU = \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} (1)(2) + (0)(0) & (1)(1) + (0)(3) \\ (4)(2) + (1)(0) & (4)(1) + (1)(3) \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 8 & 7 \end{bmatrix}$.

   Answer: L = \beginbmatrix 1 & 0 \\ 4 & 1 \endbmatrix, U = \beginbmatrix 2 & 1 \\ 0 & 3 \endbmatrix