Symmetric, diagonal, triangular, and permutation matrices — a bestiary

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Practice problems

1. Determine which of the following categories the matrix $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 5 & 0 \\ 2 & 0 & 3 \end{bmatrix}$ belongs to: diagonal, triangular, or symmetric.

2. Given the permutation matrix $P = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ and vector $v = \begin{bmatrix} 10 \\ 20 \\ 30 \end{bmatrix}$, compute Pv.

3. If $D_1 = \begin{bmatrix} 2 & 0 \\ 0 & -3 \end{bmatrix}$ and $D_2 = \begin{bmatrix} 4 & 0 \\ 0 & 5 \end{bmatrix}$, find the matrix $M = D_1 D_2 + D_1$.

Solutions

1. Determine which of the following categories the matrix $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 5 & 0 \\ 2 & 0 & 3 \end{bmatrix}$ belongs to: diagonal, triangular, or symmetric.

   - Check if it is diagonal: The entry $a_{13} = 2$ is not on the main diagonal, so it is not diagonal.
   - Check if it is triangular: There are non-zero entries both above ($a_{13} = 2$) and below ($a_{31} = 2$) the diagonal, so it is neither upper nor lower triangular.
   - Check if it is symmetric: The transpose A^T has a_{13} becoming a_{31} and vice versa. Since $a_{13} = 2$ and $a_{31} = 2$, and all other off-diagonal pairs match, $A = A^T$.

   Answer: Symmetric

2. Given the permutation matrix $P = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ and vector $v = \begin{bmatrix} 10 \\ 20 \\ 30 \end{bmatrix}$, compute Pv.

   - The first row of P is $[0, 0, 1]$, so the first entry of Pv is $0(10) + 0(20) + 1(30) = 30$.
   - The second row of P is $[1, 0, 0]$, so the second entry of Pv is $1(10) + 0(20) + 0(30) = 10$.
   - The third row of P is $[0, 1, 0]$, so the third entry of Pv is $0(10) + 1(20) + 0(30) = 20$.

   Answer: \beginbmatrix 30 \\ 10 \\ 20 \endbmatrix

3. If $D_1 = \begin{bmatrix} 2 & 0 \\ 0 & -3 \end{bmatrix}$ and $D_2 = \begin{bmatrix} 4 & 0 \\ 0 & 5 \end{bmatrix}$, find the matrix $M = D_1 D_2 + D_1$.

   - Compute $D_1 D_2$ by multiplying diagonal entries: $\begin{bmatrix} 2 \cdot 4 & 0 \\ 0 & -3 \cdot 5 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ 0 & -15 \end{bmatrix}$.
   - Add D_1 to the result: $\begin{bmatrix} 8 & 0 \\ 0 & -15 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & -3 \end{bmatrix}$.
   - Sum the corresponding entries: $\begin{bmatrix} 8+2 & 0 \\ 0 & -15-3 \end{bmatrix} = \begin{bmatrix} 10 & 0 \\ 0 & -18 \end{bmatrix}$.

   Answer: \beginbmatrix 10 & 0 \\ 0 & -18 \endbmatrix
