LU decomposition I — the idea

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Practice problems

1. Find the LU decomposition of the matrix $A = \begin{pmatrix} 2 & 1 \\ 8 & 7 \end{pmatrix}$.

2. Given $L = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$ and $U = \begin{pmatrix} 3 & 5 \\ 0 & 4 \end{pmatrix}$, find the original matrix A.

3. Find the LU decomposition of $A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 5 & 3 \\ 1 & 0 & 1 \end{pmatrix}$.

Solutions

1. Find the LU decomposition of the matrix $A = \begin{pmatrix} 2 & 1 \\ 8 & 7 \end{pmatrix}$.

   - Perform elimination on A to find U. Subtract 4 times row 1 from row 2: $8 - 4(2) = 0$ and $7 - 4(1) = 3$.
   - The resulting upper triangular matrix is $U = \begin{pmatrix} 2 & 1 \\ 0 & 3 \end{pmatrix}$.
   - The multiplier used was 4. Place this in the second row, first column of L.
   - The lower triangular matrix is $L = \begin{pmatrix} 1 & 0 \\ 4 & 1 \end{pmatrix}$.

   Answer: L = \beginpmatrix 1 & 0 \\ 4 & 1 \endpmatrix, U = \beginpmatrix 2 & 1 \\ 0 & 3 \endpmatrix

2. Given $L = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$ and $U = \begin{pmatrix} 3 & 5 \\ 0 & 4 \end{pmatrix}$, find the original matrix A.

   - Compute the product LU.
   - First row: $1(3) + 0(0) = 3$ and $1(5) + 0(4) = 5$.
   - Second row: $2(3) + 1(0) = 6$ and $2(5) + 1(4) = 14$.

   Answer: A = \beginpmatrix 3 & 5 \\ 6 & 14 \endpmatrix

3. Find the LU decomposition of $A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 5 & 3 \\ 1 & 0 & 1 \end{pmatrix}$.

   - Step 1: Eliminate column 1. $R_2 \leftarrow R_2 - 2R_1$ and $R_3 \leftarrow R_3 - 1R_1$. Multipliers are $L_{21} = 2, L_{31} = 1$.
   - Matrix becomes $\begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 0 & -2 & 0 \end{pmatrix}$.
   - Step 2: Eliminate column 2. $R_3 \leftarrow R_3 - (-2)R_2$, which is $R_3 + 2R_2$. Multiplier is $L_{32} = -2$.
   - Matrix becomes $U = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{pmatrix}$.
   - Step 3: Construct L using multipliers: $L = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & -2 & 1 \end{pmatrix}$.

   Answer: L = \beginpmatrix 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & -2 & 1 \endpmatrix, U = \beginpmatrix 1 & 2 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \endpmatrix