Elementary matrices: elimination as multiplication

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Practice problems

1. Find the elementary matrix E that performs the operation $R_2 \leftarrow R_2 - 4R_1$ on a 2×2 matrix, and verify it by computing EA for $A = \big(1 \quad 1\ 4 \quad 5\big)$.
2. Given the elementary matrix $E = \big(0 \quad 1\ 1 \quad 0\big)$, what row operation does it perform, and what is its inverse E^{-1}?
3. A 2×2 matrix A is transformed into R by first multiplying the first row by 2, then adding 3 times the first row to the second row. Write the product of elementary matrices E_2E_1 that represents this sequence.

Solutions

1. Find the elementary matrix E that performs the operation $R_2 \leftarrow R_2 - 4R_1$ on a 2×2 matrix, and verify it by computing EA for $A = \big(1 \quad 1\; 4 \quad 5\big)$.

   - Apply $R_2 \leftarrow R_2 - 4R_1$ to $I = \big(1 \quad 0\; 0 \quad 1\big)$.
   - The first row remains $\big(1 \quad 0\big)$. The second row becomes $\big(0 - 4(1) \quad 1 - 4(0)\big) = \big(-4 \quad 1\big)$.
   - So $E = \big(1 \quad 0\; -4 \quad 1\big)$.
   - Compute $EA = \big(1 \quad 0\; -4 \quad 1\big)\big(1 \quad 1\; 4 \quad 5\big) = \begin{pmatrix} 1(1) + 0(4) & 1(1) + 0(5) \\ -4(1) + 1(4) & -4(1) + 1(5) \end{pmatrix} = \big(1 \quad 1\; 0 \quad 1\big)$.

   Answer: E = \beginpmatrix 1 & 0 \ -4 & 1 \endpmatrix, E A = \beginpmatrix 1 & 1 \ 0 & 1 \endpmatrix

2. Given the elementary matrix $E = \big(0 \quad 1\; 1 \quad 0\big)$, what row operation does it perform, and what is its inverse E^{-1}?

   - Compare E to the identity matrix $I = \big(1 \quad 0\; 0 \quad 1\big)$.
   - The rows of I have been swapped. Thus, E performs the operation $R_1 \leftrightarrow R_2$.
   - To undo a swap, you swap the rows back. The operation $R_1 \leftrightarrow R_2$ is its own inverse.
   - Therefore, $E^{-1} = E = \big(0 \quad 1\; 1 \quad 0\big)$.

   Answer: Operation: R_1 \leftrightarrow R_2, E^-1 = \beginpmatrix 0 & 1 \ 1 & 0 \endpmatrix

3. A 2×2 matrix A is transformed into R by first multiplying the first row by 2, then adding 3 times the first row to the second row. Write the product of elementary matrices E_2E_1 that represents this sequence.

   - Step 1: $R_1 \leftarrow 2R_1$. The elementary matrix is $E_1 = \big(2 \quad 0\; 0 \quad 1\big)$.
   - Step 2: $R_2 \leftarrow R_2 + 3R_1$. The elementary matrix is $E_2 = \big(1 \quad 0\; 3 \quad 1\big)$.
   - The total transformation is $R = E_2(E_1A) = (E_2E_1)A$.
   - Compute $E_2E_1 = \big(1 \quad 0\; 3 \quad 1\big)\big(2 \quad 0\; 0 \quad 1\big) = \begin{pmatrix} 1(2) + 0(0) & 1(0) + 0(1) \\ 3(2) + 1(0) & 3(0) + 1(1) \end{pmatrix} = \big(2 \quad 0\; 6 \quad 1\big)$.

   Answer: E_2 E_1 = \beginpmatrix 2 & 0 \ 6 & 1 \endpmatrix
