When is a matrix invertible? First encounter with singularity

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Practice problems

1. Determine if the matrix $A = \begin{pmatrix} 3 & 6 \\ 2 & 4 \end{pmatrix}$ is singular. Explain your reasoning using the columns.
2. Find a non-zero vector x such that $Ax = 0$ for the matrix $A = \begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix}$.
3. A 3×3 matrix M has a row echelon form with only two pivots. Is M invertible? Why?

Solutions

1. Determine if the matrix $A = \begin{pmatrix} 3 & 6 \\ 2 & 4 \end{pmatrix}$ is singular. Explain your reasoning using the columns.

   - Observe the first column $c_1 = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ and the second column $c_2 = \begin{pmatrix} 6 \\ 4 \end{pmatrix}$.
   - Notice that $c_2 = 2 \cdot c_1$.
   - Since the columns are linearly dependent, the matrix is singular.

   Answer: Singular

2. Find a non-zero vector x such that $Ax = 0$ for the matrix $A = \begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix}$.

   - Set up the system $x + y = 0$ and $2x + 2y = 0$.
   - Both equations simplify to $y = -x$.
   - Pick $x = 1$, then $y = -1$.
   - The vector is $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$.

   Answer: $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

3. A 3×3 matrix M has a row echelon form with only two pivots. Is M invertible? Why?

   - A 3×3 matrix is invertible if and only if it has three pivots (one in every row/column).
   - Since M only has two pivots, the third row of the echelon form must be a row of zeros.
   - A row of zeros in the echelon form indicates that the matrix is singular.

   Answer: No, it is singular.
