Matrix inverse II — Gauss-Jordan computation

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Practice problems

1. Find the inverse of the matrix $A = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 3 \\ 2 & 5 & 2 \end{pmatrix}$ using the Gauss-Jordan method.
2. Determine if the matrix $B = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$ is invertible using the Gauss-Jordan method.
3. Given $C = \begin{pmatrix} 2 & 1 \\ 5 & 3 \end{pmatrix}$, find C^{-1} using both the 2x2 formula and the Gauss-Jordan method to verify they match.

Solutions

1. Find the inverse of the matrix $A = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 3 \\ 2 & 5 & 2 \end{pmatrix}$ using the Gauss-Jordan method.

   - Set up augmented matrix $[A|I]$.
   - R3 ¡- R3 - 2*R1 results in $\left[\begin{smallmatrix} 1 & 2 & 1 & | & 1 & 0 & 0 \\ 0 & 1 & 3 & | & 0 & 1 & 0 \\ 0 & 1 & 0 & | & -2 & 0 & 1 \end{smallmatrix}\right]$.
   - R3 ¡- R3 - R2 results in $\left[\begin{smallmatrix} 1 & 2 & 1 & | & 1 & 0 & 0 \\ 0 & 1 & 3 & | & 0 & 1 & 0 \\ 0 & 0 & -3 & | & -2 & -1 & 1 \end{smallmatrix}\right]$.
   - R3 ¡- -1/3 * R3 results in $\left[\begin{smallmatrix} 1 & 2 & 1 & | & 1 & 0 & 0 \\ 0 & 1 & 3 & | & 0 & 1 & 0 \\ 0 & 0 & 1 & | & 2/3 & 1/3 & -1/3 \end{smallmatrix}\right]$.
   - R2 ¡- R2 - 3*R3 and R1 ¡- R1 - R3 results in $\left[\begin{smallmatrix} 1 & 2 & 0 & | & 1/3 & -1/3 & 1/3 \\ 0 & 1 & 0 & | & -2 & 0 & 1 \\ 0 & 0 & 1 & | & 2/3 & 1/3 & -1/3 \end{smallmatrix}\right]$.
   - R1 ¡- R1 - 2*R2 results in $\left[\begin{smallmatrix} 1 & 0 & 0 & | & 13/3 & -1/3 & -5/3 \\ 0 & 1 & 0 & | & -2 & 0 & 1 \\ 0 & 0 & 1 & | & 2/3 & 1/3 & -1/3 \end{smallmatrix}\right]$.

   Answer: \beginpmatrix 13/3 & -1/3 & -5/3 \\ -2 & 0 & 1 \\ 2/3 & 1/3 & -1/3 \endpmatrix

2. Determine if the matrix $B = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$ is invertible using the Gauss-Jordan method.

   - Set up augmented matrix $[B|I]$.
   - R2 ¡- R2 - 4*R1 and R3 ¡- R3 - 7*R1 results in $\left[\begin{smallmatrix} 1 & 2 & 3 & | & 1 & 0 & 0 \\ 0 & -3 & -6 & | & -4 & 1 & 0 \\ 0 & -6 & -12 & | & -7 & 0 & 1 \end{smallmatrix}\right]$.
   - R3 ¡- R3 - 2*R2 results in $\left[\begin{smallmatrix} 1 & 2 & 3 & | & 1 & 0 & 0 \\ 0 & -3 & -6 & | & -4 & 1 & 0 \\ 0 & 0 & 0 & | & 1 & -2 & 1 \end{smallmatrix}\right]$.
   - The left side contains a row of zeros, meaning the matrix is singular.

   Answer: Not invertible (singular)

3. Given $C = \begin{pmatrix} 2 & 1 \\ 5 & 3 \end{pmatrix}$, find C^{-1} using both the 2x2 formula and the Gauss-Jordan method to verify they match.

   - Formula: $\det(C) = (2)(3) - (1)(5) = 1$. $C^{-1} = \frac{1}{1}\begin{pmatrix} 3 & -1 \\ -5 & 2 \end{pmatrix} = \begin{pmatrix} 3 & -1 \\ -5 & 2 \end{pmatrix}$.
   - Gauss-Jordan: $\left[\begin{smallmatrix} 2 & 1 & | & 1 & 0 \\ 5 & 3 & | & 0 & 1 \end{smallmatrix}\right]$.
   - R1 ¡- 0.5*R1 results in $\left[\begin{smallmatrix} 1 & 0.5 & | & 0.5 & 0 \\ 5 & 3 & | & 0 & 1 \end{smallmatrix}\right]$.
   - R2 ¡- R2 - 5*R1 results in $\left[\begin{smallmatrix} 1 & 0.5 & | & 0.5 & 0 \\ 0 & 0.5 & | & -2.5 & 1 \end{smallmatrix}\right]$.
   - R2 ¡- 2*R2 results in $\left[\begin{smallmatrix} 1 & 0.5 & | & 0.5 & 0 \\ 0 & 1 & | & -5 & 2 \end{smallmatrix}\right]$.
   - R1 ¡- R1 - 0.5*R2 results in $\left[\begin{smallmatrix} 1 & 0 & | & 3 & -1 \\ 0 & 1 & | & -5 & 2 \end{smallmatrix}\right]$.

   Answer: \beginpmatrix 3 & -1 \\ -5 & 2 \endpmatrix