Matrix inverse I — concept and 2x2 formula

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Practice problems

1. Find the inverse of the matrix $A = \begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix}$.
2. Determine if the matrix $B = \begin{pmatrix} 2 & 4 \\ 1 & 2 \end{pmatrix}$ is invertible. If so, find its inverse.
3. Find the inverse of $C = \begin{pmatrix} 4 & 3 \\ 3 & 2 \end{pmatrix}$ and verify it by computing CC^{-1}.

Solutions

1. Find the inverse of the matrix $A = \begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix}$.

   - Calculate the determinant: $\det(A) = (3)(2) - (1)(5) = 6 - 5 = 1$.
   - Swap the main diagonal elements (3 and 2) and negate the off-diagonal elements (1 and 5).
   - The resulting matrix is $\begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix}$.
   - Multiply by $1/\det(A) = 1/1$.

   Answer: \beginpmatrix 2 & -1 \\ -5 & 3 \endpmatrix

2. Determine if the matrix $B = \begin{pmatrix} 2 & 4 \\ 1 & 2 \end{pmatrix}$ is invertible. If so, find its inverse.

   - Calculate the determinant: $\det(B) = (2)(2) - (4)(1) = 4 - 4 = 0$.
   - Since the determinant is zero, the matrix is singular.

   Answer: Not invertible (singular)

3. Find the inverse of $C = \begin{pmatrix} 4 & 3 \\ 3 & 2 \end{pmatrix}$ and verify it by computing CC^{-1}.

   - Calculate the determinant: $\det(C) = (4)(2) - (3)(3) = 8 - 9 = -1$.
   - Apply the formula: $C^{-1} = \frac{1}{-1}\begin{pmatrix} 2 & -3 \\ -3 & 4 \end{pmatrix} = \begin{pmatrix} -2 & 3 \\ 3 & -4 \end{pmatrix}$.
   - Verify: $\begin{pmatrix} 4 & 3 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} -2 & 3 \\ 3 & -4 \end{pmatrix} = \begin{pmatrix} -8+9 & 12-12 \\ -6+6 & 9-8 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

   Answer: \beginpmatrix -2 & 3 \\ 3 & -4 \endpmatrix
