The identity matrix and matrix powers

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Practice problems

1. Compute A^2 for the matrix $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$.
2. Let $A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. Compute A^2 and A^4.
3. If A is a 3×3 matrix such that $A^2 = I$, what is the value of $A^3 + 2A^2 + A$ in terms of A and I?

Solutions

1. Compute A^2 for the matrix $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$.

   - The top left element is the dot product of the first row $(1, 2)$ and first column $(1, 3)$, which is $1(1) + 2(3) = 7$.
   - The top right element is the dot product of the first row $(1, 2)$ and second column $(2, 4)$, which is $1(2) + 2(4) = 10$.
   - The bottom left element is the dot product of the second row $(3, 4)$ and first column $(1, 3)$, which is $3(1) + 4(3) = 15$.
   - The bottom right element is the dot product of the second row $(3, 4)$ and second column $(2, 4)$, which is $3(2) + 4(4) = 22$.

   Answer: \beginpmatrix 7 & 10 \\ 15 & 22 \endpmatrix

2. Let $A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. Compute A^2 and A^4.

   - Compute A^2: the top left is $0(0) + (-1)(1) = -1$; top right is $0(-1) + (-1)(0) = 0$; bottom left is $1(0) + 0(1) = 0$; bottom right is $1(-1) + 0(0) = -1$. So $A^2 = -I$.
   - Compute A^4 as $(A^2)^2$. Since $A^2 = -I$, then $A^4 = (-I)(-I)$.
   - Multiplying $-I$ by $-I$ is the same as $(-1)(-1)I^2$, which is $1 \times I = I$.

   Answer: Aˆ2 = \beginpmatrix -1 & 0 \\ 0 & -1 \endpmatrix, Aˆ4 = \beginpmatrix 1 & 0 \\ 0 & 1 \endpmatrix

3. If A is a 3×3 matrix such that $A^2 = I$, what is the value of $A^3 + 2A^2 + A$ in terms of A and I?

   - Substitute $A^2 = I$ into the expression: $A^3 + 2A^2 + A = A(A^2) + 2(I) + A$.
   - Since $A^2 = I$, the first term $A(A^2)$ becomes $A(I) = A$.
   - The expression is now $A + 2I + A$.
   - Combine the A terms to get $2A + 2I$.

   Answer: 2A + 2I
