Matrix multiplication properties; why AB is not BA

Lesson 32 · Module 2 — Matrix Algebra · Linear Algebra · dailymaths.uk/linalg

Practice problems

1. Given $A = \begin{pmatrix}1 & 2\ 3 & 4\end{pmatrix}$ and $B = \begin{pmatrix}0 & 1\ 1 & 0\end{pmatrix}$, compute AB and BA to show that $AB \neq BA$.
2. Let $A = \begin{pmatrix}1 & 0\ 0 & 1\end{pmatrix}$ and $B = \begin{pmatrix}5 & 6\ 7 & 8\end{pmatrix}$. Compute AB and BA. Does $AB = BA$ in this specific case?
3. Simplify the expression $A(B + C) - AB$, where A, B, C are 2×2 matrices.

Solutions

1. Given $A = \begin{pmatrix} 1 & 2 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 1 1 & 0 \end{pmatrix}$, compute AB and BA to show that $AB \neq BA$.

   - Compute AB: the first row is $(1 \cdot 0 + 2 \cdot 1, 1 \cdot 1 + 2 \cdot 0) = (2, 1)$, the second row is $(3 \cdot 0 + 4 \cdot 1, 3 \cdot 1 + 4 \cdot 0) = (4, 3)$. So $AB = \begin{pmatrix} 2 & 1 4 & 3 \end{pmatrix}$.
   - Compute BA: the first row is $(0 \cdot 1 + 1 \cdot 3, 0 \cdot 2 + 1 \cdot 4) = (3, 4)$, the second row is $(1 \cdot 1 + 0 \cdot 3, 1 \cdot 2 + 0 \cdot 4) = (1, 2)$. So $BA = \begin{pmatrix} 3 & 4 1 & 2 \end{pmatrix}$.
   - Compare the results: $\begin{pmatrix} 2 & 1 4 & 3 \end{pmatrix} \neq \begin{pmatrix} 3 & 4 1 & 2 \end{pmatrix}$.

   Answer: AB = \beginpmatrix 2 & 1 \ 4 & 3 \endpmatrix, BA = \beginpmatrix 3 & 4 \ 1 & 2 \endpmatrix

2. Let $A = \begin{pmatrix} 1 & 0 0 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 5 & 6 7 & 8 \end{pmatrix}$. Compute AB and BA. Does $AB = BA$ in this specific case?

   - Compute AB: the first row is $(1 \cdot 5 + 0 \cdot 7, 1 \cdot 6 + 0 \cdot 8) = (5, 6)$, the second row is $(0 \cdot 5 + 1 \cdot 7, 0 \cdot 6 + 1 \cdot 8) = (7, 8)$. So $AB = \begin{pmatrix} 5 & 6 7 & 8 \end{pmatrix}$.
   - Compute BA: the first row is $(5 \cdot 1 + 6 \cdot 0, 5 \cdot 0 + 6 \cdot 1) = (5, 6)$, the second row is $(7 \cdot 1 + 8 \cdot 0, 7 \cdot 0 + 8 \cdot 1) = (7, 8)$. So $BA = \begin{pmatrix} 5 & 6 7 & 8 \end{pmatrix}$.
   - Compare the results: $AB = BA$.

   Answer: Yes, AB = BA = \beginpmatrix 5 & 6 \ 7 & 8 \endpmatrix

3. Simplify the expression $A(B + C) - AB$, where A, B, C are 2×2 matrices.

   - Apply the distributive law: $A(B + C) = AB + AC$.
   - Substitute this back into the expression: $(AB + AC) - AB$.
   - Rearrange the terms: $AB - AB + AC$.
   - The AB terms cancel out, leaving AC.

   Answer: AC
