Matrix multiplication II — why it's defined that way

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Practice problems

1. Let $R = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ be a 90° rotation and $S = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$ be a scaling by 3. Find the matrix C that represents rotating a vector first and then scaling the result.
2. Given $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$, compute AB and BA. Do they commute?
3. A transformation T_1 is represented by $M_1 = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$ and T_2 is represented by $M_2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$. If we apply T_1 then T_2, what is the final image of the vector $\mathbf{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$? Find the composition matrix first.

Solutions

1. Let $R = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ be a 90° rotation and $S = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$ be a scaling by 3. Find the matrix C that represents rotating a vector first and then scaling the result.

   - The vector is rotated first, so R is the rightmost matrix.
   - The vector is then scaled, so S is the leftmost matrix.
   - We compute $C = SR$.
   - The first row of S dotted with the first column of R is $3(0) + 0(1) = 0$.
   - The first row of S dotted with the second column of R is $3(-1) + 0(0) = -3$.
   - The second row of S dotted with the first column of R is $0(0) + 3(1) = 3$.
   - The second row of S dotted with the second column of R is $0(-1) + 3(0) = 0$.

   Answer: \beginbmatrix 0 & -3 \\ 3 & 0 \endbmatrix

2. Given $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$, compute AB and BA. Do they commute?

   - Compute AB: row 1 of A dot col 1 of B is $1(1) + 2(3) = 7$; row 1 of A dot col 2 of B is $1(0) + 2(1) = 2$; row 2 of A dot col 1 of B is $0(1) + 1(3) = 3$; row 2 of A dot col 2 of B is $0(0) + 1(1) = 1$. So $AB = \begin{bmatrix} 7 & 2 \\ 3 & 1 \end{bmatrix}$.
   - Compute BA: row 1 of B dot col 1 of A is $1(1) + 0(0) = 1$; row 1 of B dot col 2 of A is $1(2) + 0(1) = 2$; row 2 of B dot col 1 of A is $3(1) + 1(0) = 3$; row 2 of B dot col 2 of A is $3(2) + 1(1) = 7$. So $BA = \begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}$.
   - Since $AB \neq BA$, the matrices do not commute.

   Answer: AB = \beginbmatrix 7 & 2 \\ 3 & 1 \endbmatrix, BA = \beginbmatrix 1 & 2 \\ 3 & 7 \endbmatrix, \text No they do not commute.

3. A transformation T_1 is represented by $M_1 = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$ and T_2 is represented by $M_2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$. If we apply T_1 then T_2, what is the final image of the vector $\mathbf{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$? Find the composition matrix first.

   - The sequence is T_1 then T_2, so the composition matrix is $C = M_2 M_1$.
   - Compute $C = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1(2) + 1(0) & 1(0) + 1(1) \\ 0(2) + 1(0) & 0(0) + 1(1) \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$.
   - Now apply C to $\mathbf{v}$: $\begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2(1) + 1(1) \\ 0(1) + 1(1) \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$.

   Answer: \beginbmatrix 3 \\ 1 \endbmatrix