Matrix multiplication I — definition and mechanics

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Practice problems

1. Compute the product $C = AB$ where $A = \begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ 2 & 5 \end{bmatrix}$.
2. Given $A = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$, compute AB and BA to show they are different.
3. Let A be a 3×2 matrix and B be a 2×4 matrix. What are the dimensions of the product $C = AB$? Is the product BA defined? Explain.

Solutions

1. Compute the product $C = AB$ where $A = \begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ 2 & 5 \end{bmatrix}$.

   - Check dimensions: $(2 \times 2) \times (2 \times 2)$ is defined and results in a 2×2 matrix.
   - Compute $c_{11} = (2)(1) + (-1)(2) = 2 - 2 = 0$.
   - Compute $c_{12} = (2)(4) + (-1)(5) = 8 - 5 = 3$.
   - Compute $c_{21} = (0)(1) + (3)(2) = 0 + 6 = 6$.
   - Compute $c_{22} = (0)(4) + (3)(5) = 0 + 15 = 15$.

   Answer: \beginbmatrix 0 & 3 \\ 6 & 15 \endbmatrix

2. Given $A = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$, compute AB and BA to show they are different.

   - Compute AB: $c_{11} = 1(1) + 2(0) = 1$, $c_{12} = 1(1) + 2(2) = 5$, $c_{21} = 3(1) + 0(0) = 3$, $c_{22} = 3(1) + 0(2) = 3$. So $AB = \begin{bmatrix} 1 & 5 \\ 3 & 3 \end{bmatrix}$.
   - Compute BA: $c_{11} = 1(1) + 1(3) = 4$, $c_{12} = 1(2) + 1(0) = 2$, $c_{21} = 0(1) + 2(3) = 6$, $c_{22} = 0(2) + 2(0) = 0$. So $BA = \begin{bmatrix} 4 & 2 \\ 6 & 0 \end{bmatrix}$.
   - Compare the results: $\begin{bmatrix} 1 & 5 \\ 3 & 3 \end{bmatrix} \neq \begin{bmatrix} 4 & 2 \\ 6 & 0 \end{bmatrix}$.

   Answer: AB = \beginbmatrix 1 & 5 \\ 3 & 3 \endbmatrix, BA = \beginbmatrix 4 & 2 \\ 6 & 0 \endbmatrix

3. Let A be a 3×2 matrix and B be a 2×4 matrix. What are the dimensions of the product $C = AB$? Is the product BA defined? Explain.

   - For $C = AB$, the dimensions are $(3 \times 2) \times (2 \times 4)$. The inner dimensions match (2), so the product is defined.
   - The resulting dimensions are the outer dimensions: 3×4.
   - For BA, the dimensions are $(2 \times 4) \times (3 \times 2)$.
   - The inner dimensions are 4 and 3. Since $4 \neq 3$, the product BA is undefined.

   Answer: C is 3×4; BA is undefined.
