Matrix-vector multiplication: two interpretations

Practice problems

1. Compute the product of the matrix $A = \begin{bmatrix} 1 & 0 & 2 \\ -1 & 3 & 1 \end{bmatrix}$ and the vector $\mathbf{x} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$.

2. Given the matrix $B = \begin{bmatrix} 2 & 1 \\ 0 & -2 \\ 3 & 4 \end{bmatrix}$ and the vector $\mathbf{v} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, find $B\mathbf{v}$ using the column view.

3. If a matrix C has dimensions 4×3 and a vector $\mathbf{w}$ has dimensions 3×1, what are the dimensions of the resulting vector $C\mathbf{w}$? If $\mathbf{w}$ had 4 components instead, would the multiplication be possible?

Solutions

1. Compute the product of the matrix $A = \begin{bmatrix} 1 & 0 & 2 \\ -1 & 3 & 1 \end{bmatrix}$ and the vector $\mathbf{x} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$.

   - Using the row view, the first component is $1(2) + 0(1) + 2(3) = 2 + 0 + 6 = 8$.
   - The second component is $-1(2) + 3(1) + 1(3) = -2 + 3 + 3 = 4$.
   - The resulting vector is $\begin{bmatrix} 8 \\ 4 \end{bmatrix}$.

   Answer: \beginbmatrix 8 \\ 4 \endbmatrix

2. Given the matrix $B = \begin{bmatrix} 2 & 1 \\ 0 & -2 \\ 3 & 4 \end{bmatrix}$ and the vector $\mathbf{v} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, find $B\mathbf{v}$ using the column view.

   - The column view is $1 \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix} + 2 \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix}$.
   - This equals $\begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix} + \begin{bmatrix} 2 \\ -4 \\ 8 \end{bmatrix}$.
   - Summing the components gives $\begin{bmatrix} 4 \\ -4 \\ 11 \end{bmatrix}$.

   Answer: \beginbmatrix 4 \\ -4 \\ 11 \endbmatrix

3. If a matrix C has dimensions 4×3 and a vector $\mathbf{w}$ has dimensions 3×1, what are the dimensions of the resulting vector $C\mathbf{w}$? If $\mathbf{w}$ had 4 components instead, would the multiplication be possible?

   - The product of a 4×3 matrix and a 3×1 vector results in a 4×1 vector.
   - For multiplication to be possible, the number of columns in the matrix must match the number of components in the vector.
   - Since the matrix has 3 columns, a vector with 4 components would not match, making the multiplication impossible.

   Answer: 4 \times 1; No
