

Module 1 review + self-test

Lesson 27 · Module 1 — Systems of Linear Equations · Linear Algebra · dailymaths.uk/linalg

Practice problems

1. Solve the system represented by the augmented matrix $\left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 0 & 1 & -1 & 1 \\ 2 & 5 & 2 & 9 \end{array}\right]$.
2. Determine if the system $\left[\begin{array}{cc|c} 1 & 1 & 2 \\ 2 & 2 & 5 \end{array}\right]$ is consistent.
3. Find the general solution for $\left[\begin{array}{cc|c} 1 & 3 & 5 \\ 0 & 0 & 0 \end{array}\right]$.

Solutions

1. Solve the system represented by the augmented matrix $\left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 0 & 1 & -1 & 1 \\ 2 & 5 & 2 & 9 \end{array} \right]$.

- Step 1: Clear the first column. $R_3 \leftarrow R_3 - 2R_1$ gives $\left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 0 & 1 & -1 & 1 \\ 0 & 1 & 0 & 1 \end{array} \right]$.
- Step 2: Clear the second column. $R_3 \leftarrow R_3 - R_2$ gives $\left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 0 \end{array} \right]$.
- Step 3: Back substitute. From R_3 , $z = 0$.
- Step 4: From R_2 , $y - 0 = 1$, so $y = 1$.
- Step 5: From R_1 , $x + 2(1) + 0 = 4$, so $x = 2$.

Answer: (2, 1, 0)

2. Determine if the system $\left[\begin{array}{cc|c} 1 & 1 & 2 \\ 2 & 2 & 5 \end{array} \right]$ is consistent.

- Step 1: Perform $R_2 \leftarrow R_2 - 2R_1$.
- Step 2: The resulting matrix is $\left[\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & 0 & 1 \end{array} \right]$.
- Step 3: The second row implies $0 = 1$, which is a contradiction.

Answer: Inconsistent (no solution)

3. Find the general solution for $\left[\begin{array}{cc|c} 1 & 3 & 5 \\ 0 & 0 & 0 \end{array} \right]$.

- Step 1: Identify pivots. The only pivot is in the first column.
- Step 2: Identify free variables. The second column has no pivot, so y is free.
- Step 3: Express x in terms of y . $x + 3y = 5 \implies x = 5 - 3y$.

Answer: $x = 5 - 3y$, y is free