Elimination as matrix multiplication — a first look ahead

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Practice problems

1. Find the 3×3 elementary matrix E that performs the operation $R_3 \leftarrow R_3 - 4R_2$ on a matrix A.
2. Given $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, find the result of EA where $E = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.
3. If E_1 scales row 1 by 5 and E_2 replaces row 2 with $R_2 + 2R_1$, what is the single matrix $E_{total} = E_2 E_1$ for a 2×2 system?

Solutions

1. Find the 3×3 elementary matrix E that performs the operation $R_3 \leftarrow R_3 - 4R_2$ on a matrix A.

   - Start with the 3×3 identity matrix I.
   - The operation affects row 3 using row 2.
   - Place the multiplier -4 in the position corresponding to row 3, column 2.
   - Keep all other entries of the identity matrix the same.

   Answer: \beginpmatrix 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -4 & 1 \endpmatrix

2. Given $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, find the result of EA where $E = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

   - Recognize that E is the elementary matrix for swapping row 1 and row 2.
   - Apply the swap to the rows of A.
   - The first row of A becomes the second row of the result.
   - The second row of A becomes the first row of the result.

   Answer: \beginpmatrix 3 & 4 \\ 1 & 2 \endpmatrix

3. If E_1 scales row 1 by 5 and E_2 replaces row 2 with $R_2 + 2R_1$, what is the single matrix $E_{total} = E_2 E_1$ for a 2×2 system?

   - Write $E_1 = \begin{pmatrix} 5 & 0 \\ 0 & 1 \end{pmatrix}$.
   - Write $E_2 = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$.
   - Compute the product $E_2 E_1$.
   - Top row: $(1 \cdot 5) + (0 \cdot 0) = 5$ and $(1 \cdot 0) + (0 \cdot 1) = 0$.
   - Bottom row: $(2 \cdot 5) + (1 \cdot 0) = 10$ and $(2 \cdot 0) + (1 \cdot 1) = 1$.

   Answer: \beginpmatrix 5 & 0 \\ 10 & 1 \endpmatrix