Applications II: balancing chemical equations, fitting polynomials

Lesson 25 · Module 1 — Systems of Linear Equations · Linear Algebra · dailymaths.uk/linalg

Practice problems

1. Find the quadratic polynomial $y = ax^2 + bx + c$ that passes through the points $(0, 1)$, $(1, 3)$, and $(2, 7)$.
2. A chemical reaction is represented by the unbalanced equation: $x_1\mathrm{C_3H_8} + x_2\mathrm{O_2} \to x_3\mathrm{C_3H_6O_3} + x_4\mathrm{H_2O}$. Set up the homogeneous system of equations for Carbon, Hydrogen, and Oxygen.
3. Determine if a unique parabola $y = ax^2 + bx + c$ can be fit through the points $(1, 2)$, $(2, 4)$, and $(1, 5)$. Explain why or why not using linear algebra concepts.

Solutions

1. Find the quadratic polynomial $y = ax^2 + bx + c$ that passes through the points $(0, 1)$, $(1, 3)$, and $(2, 7)$.

   - Plug in $(0, 1)$: $a(0)^2 + b(0) + c = 1 \implies c = 1$.
   - Plug in $(1, 3)$: $a(1)^2 + b(1) + 1 = 3 \implies a + b = 2$.
   - Plug in $(2, 7)$: $a(2)^2 + b(2) + 1 = 7 \implies 4a + 2b = 6$.
   - Divide the last equation by 2: $2a + b = 3$.
   - Subtract $(a + b = 2)$ from $(2a + b = 3)$ to get $a = 1$.
   - Substitute $a = 1$ into $a + b = 2$ to get $b = 1$.

   Answer: y = xˆ2 + x + 1

2. A chemical reaction is represented by the unbalanced equation: $x_1C_3H_8 + x_2O_2 \to x_3C_3H_6O_3 + x_4H_2O$. Set up the homogeneous system of equations for Carbon, Hydrogen, and Oxygen.

   - Carbon: $3x_1 = 3x_3 \implies 3x_1 - 3x_3 = 0$.
   - Hydrogen: $8x_1 = 6x_3 + 2x_4 \implies 8x_1 - 6x_3 - 2x_4 = 0$.
   - Oxygen: $2x_2 = 3x_3 + x_4 \implies 2x_2 - 3x_3 - x_4 = 0$.

   Answer: 3x_1 - 3x_3 = 0, 8x_1 - 6x_3 - 2x_4 = 0, 2x_2 - 3x_3 - x_4 = 0

3. Determine if a unique parabola $y = ax^2 + bx + c$ can be fit through the points $(1, 2)$, $(2, 4)$, and $(1, 5)$. Explain why or why not using linear algebra concepts.

   - The points $(1, 2)$ and $(1, 5)$ have the same x-coordinate but different y-coordinates.
   - This means for $x = 1$, the system requires $a + b + c = 2$ and $a + b + c = 5$.
   - Subtracting these equations gives $0 = 3$, which is a contradiction.
   - The system is inconsistent, meaning no such parabola exists.

   Answer: No, the system is inconsistent because the points $(1, 2)$ and $(1, 5)$ violate the vertical line test for functions.

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