Applications I: network flows and circuits

Lesson 24 · Module 1 — Systems of Linear Equations · Linear Algebra · dailymaths.uk/linalg

Practice problems

1. A network has three nodes. Node 1: external input of 8, flows x_1 and x_2 leave. Node 2: x_1 enters, x_3 leaves. Node 3: x_2 and x_3 enter, external output of 8. Find the general solution.

2. In a circuit junction, current I_1 and I_2 enter, while I_3 and I_4 leave. If $I_1 = 5\text{A}$ and $I_2 = 3\text{A}$, and we know $I_3 = 2I_4$, find the values of I_3 and I_4.

3. A network has four nodes. Node 1: input 10, flows x_1, x_2 leave. Node 2: x_1 enters, x_3, x_4 leave. Node 3: x_2, x_3 enter, x_5 leaves. Node 4: x_4, x_5 enter, output 10. Determine if the system has a unique solution or free variables.

Solutions

1. A network has three nodes. Node 1: external input of 8, flows x_1 and x_2 leave. Node 2: x_1 enters, x_3 leaves. Node 3: x_2 and x_3 enter, external output of 8. Find the general solution.

   - Node 1: $x_1 + x_2 = 8$
   - Node 2: $x_1 = x_3$
   - Node 3: $x_2 + x_3 = 8$
   - The third equation is redundant. From node 2, $x_1 = x_3$. Substituting into node 1, $x_3 + x_2 = 8$, which matches node 3.
   - Let $x_3 = t$. Then $x_1 = t$ and $x_2 = 8 - t$.

   Answer: $x_1 = t, x_2 = 8 - t, x_3 = t$

2. In a circuit junction, current I_1 and I_2 enter, while I_3 and I_4 leave. If $I_1 = 5\text{A}$ and $I_2 = 3\text{A}$, and we know $I_3 = 2I_4$, find the values of I_3 and I_4.

   - Conservation at node: $I_1 + I_2 = I_3 + I_4$
   - Substitute knowns: $5 + 3 = I_3 + I_4 \implies I_3 + I_4 = 8$
   - Substitute constraint: $2I_4 + I_4 = 8 \implies 3I_4 = 8$
   - Solve for I_4: $I_4 = 8/3\text{A}$
   - Solve for I_3: $I_3 = 2(8/3) = 16/3\text{A}$

   Answer: $I_3 = 16/3\text{A}, I_4 = 8/3\text{A}$

3. A network has four nodes. Node 1: input 10, flows x_1, x_2 leave. Node 2: x_1 enters, x_3, x_4 leave. Node 3: x_2, x_3 enter, x_5 leaves. Node 4: x_4, x_5 enter, output 10. Determine if the system has a unique solution or free variables.

   - Eq 1: $x_1 + x_2 = 10$
   - Eq 2: $x_1 - x_3 - x_4 = 0$
   - Eq 3: $x_2 + x_3 - x_5 = 0$
   - Eq 4: $x_4 + x_5 = 10$
   - Matrix: $[[1, 1, 0, 0, 0, 10], [1, 0, -1, -1, 0, 0], [0, 1, 1, 0, -1, 0], [0, 0, 0, 1, 1, 10]]$.
   - Row reduction shows the 4th equation is a linear combination of the first three.
   - With 5 variables and 3 independent equations, there are $5 - 3 = 2$ free variables.

   Answer: The system has infinitely many solutions with 2 free variables.
