

Elimination clinic — mixed practice

Lesson 23 · Module 1 — Systems of Linear Equations · Linear Algebra · dailymaths.uk/linalg

Practice problems

1. Solve the system: $x + 2y + z = 4$, $3x - y + 2z = 1$, and $4x + y + 3z = 5$.
2. Determine if the system $x + y = 2$ and $2x + 2y = 5$ is consistent.
3. Find the non-trivial solutions to the homogeneous system $x - 2y + z = 0$ and $2x - 4y + 2z = 0$.

Solutions

1. Solve the system: $x + 2y + z = 4$, $3x - y + 2z = 1$, and $4x + y + 3z = 5$.

- Set up the augmented matrix $[[1, 2, 1, 4], [3, -1, 2, 1], [4, 1, 3, 5]]$.
- Perform $R2 \leftarrow R2 - 3R1$ and $R3 \leftarrow R3 - 4R1$ to get $[[1, 2, 1, 4], [0, -7, -1, -11], [0, -7, -1, -11]]$.
- Perform $R3 \leftarrow R3 - R2$ to get $[[1, 2, 1, 4], [0, -7, -1, -11], [0, 0, 0, 0]]$.
- Identify z as a free variable. Let $z = t$.
- From $R2$, $-7y - t = -11$, so $y = (11 - t)/7$.
- From $R1$, $x = 4 - 2y - z = 4 - 2(11 - t)/7 - t = (28 - 22 + 2t - 7t)/7 = (6 - 5t)/7$.

Answer: $\mathbf{x} = \begin{bmatrix} 6/7 \\ 11/7 \\ 0 \end{bmatrix} + t \begin{bmatrix} -5/7 \\ -1/7 \\ 1 \end{bmatrix}$

2. Determine if the system $x + y = 2$ and $2x + 2y = 5$ is consistent.

- Set up the augmented matrix $[[1, 1, 2], [2, 2, 5]]$.
- Perform $R2 \leftarrow R2 - 2R1$.
- The resulting matrix is $[[1, 1, 2], [0, 0, 1]]$.
- The second row represents the equation $0 = 1$, which is a contradiction.

Answer: Inconsistent (No solution)

3. Find the non-trivial solutions to the homogeneous system $x - 2y + z = 0$ and $2x - 4y + 2z = 0$.

- Set up the coefficient matrix $[[1, -2, 1], [2, -4, 2]]$.
- Perform $R2 \leftarrow R2 - 2R1$ to get $[[1, -2, 1], [0, 0, 0]]$.
- Variables y and z are free. Let $y = s$ and $z = t$.
- Then $x = 2s - t$.

Answer: $\mathbf{x} = s \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$