Inconsistent systems: recognising no-solution cases

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Practice problems

1. Determine if the following system is consistent or inconsistent: $x + 3y = 4$ and $2x + 6y = 10$.
2. A 3×3 system has an augmented matrix in RREF where the third row is $[0\ 0\ 0 \mid 1]$. Is the system consistent? Explain why.
3. Consider the system $x + y = 2$, $y + z = 2$, and $x + z = 2$. Is this system consistent? If so, find the solution.

Solutions

1. Determine if the following system is consistent or inconsistent: $x + 3y = 4$ and $2x + 6y = 10$.

   - Write the augmented matrix: $\left[\begin{array}{cc|c} 1 & 3 & 4 \\ 2 & 6 & 10 \end{array}\right]$
   - Perform $R_2 \leftarrow R_2 - 2R_1$: $2 - 2(1) = 0$, $6 - 2(3) = 0$, $10 - 2(4) = 2$.
   - The resulting matrix is $\left[\begin{array}{cc|c} 1 & 3 & 4 \\ 0 & 0 & 2 \end{array}\right]$.
   - The second row represents $0 = 2$, which is a contradiction.

   Answer: Inconsistent

2. A 3×3 system has an augmented matrix in RREF where the third row is $[0\ 0\ 0 \mid 1]$. Is the system consistent? Explain why.

   - The row $[0\ 0\ 0 \mid 1]$ translates to the equation $0x + 0y + 0z = 1$.
   - This simplifies to $0 = 1$.
   - Since 0 can never equal 1, there are no values of x, y, z that satisfy this equation.

   Answer: Inconsistent

3. Consider the system $x + y = 2$, $y + z = 2$, and $x + z = 2$. Is this system consistent? If so, find the solution.

   - Augmented matrix: $\left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & 1 & 1 & 2 \\ 1 & 0 & 1 & 2 \end{array}\right]$
   - Perform $R_3 \leftarrow R_3 - R_1$: $\left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & 1 & 1 & 2 \\ 0 & -1 & 1 & 0 \end{array}\right]$
   - Perform $R_3 \leftarrow R_3 + R_2$: $\left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 2 & 2 \end{array}\right]$
   - The system is consistent because there is no row of the form $[0\ 0\ 0 \mid k]$.
   - Back-substitute: $2z = 2 \implies z = 1$. Then $y + 1 = 2 \implies y = 1$. Then $x + 1 = 2 \implies x = 1$.

   Answer: Consistent; solution is $(1, 1, 1)$