

Free variables and infinite solution families

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Practice problems

1. Find the parametric solution for the system represented by the equation $2x + 3y = 12$.
2. Given the RREF matrix $\begin{pmatrix} 1 & 2 & 0 & 4 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 \end{pmatrix}$, find the general solution in parametric vector form.
3. A system of 3 equations with 5 variables is consistent and has 2 pivots. What is the dimension of the solution set, and how many parameters are needed to describe it?

Solutions

1. Find the parametric solution for the system represented by the equation $2x + 3y = 12$.

- Identify that there is one equation and two variables, so one variable must be free.
- Let $y = t$.
- Solve for x : $2x = 12 - 3t$, so $x = 6 - 1.5t$.
- Write in vector form: $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6 \\ 0 \end{pmatrix} + t \begin{pmatrix} -1.5 \\ 1 \end{pmatrix}$.

Answer: $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6 \\ 0 \end{pmatrix} + t \begin{pmatrix} -1.5 \\ 1 \end{pmatrix}$

2. Given the RREF matrix $\begin{pmatrix} 1 & 2 & 0 & 4 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 \end{pmatrix}$, find the general solution in parametric vector form.

- Pivots are in columns 1 and 3. Column 2 is free.
- Let $x_2 = t$.
- From row 2: $x_3 = 5$.
- From row 1: $x_1 + 2t = 4$, so $x_1 = 4 - 2t$.
- The solution vector is $\begin{pmatrix} 4 - 2t \\ t \\ 5 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ 5 \end{pmatrix} + t \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$.

Answer: $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ 5 \end{pmatrix} + t \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$

3. A system of 3 equations with 5 variables is consistent and has 2 pivots. What is the dimension of the solution set, and how many parameters are needed to describe it?

- The number of variables is 5.
- The number of pivots is 2.
- The number of free variables is $5 - 2 = 3$.
- The dimension of the solution set is equal to the number of free variables.

Answer: Dimension is 3; 3 parameters are needed.