

Row echelon form and reduced row echelon form

Lesson 19 · Module 1 — Systems of Linear Equations · Linear Algebra · dailymaths.uk/linalg

Practice problems

1. Transform the following augmented matrix into Reduced Row Echelon Form: $\left(\begin{array}{cc|c} 1 & 3 & 7 \\ 2 & 8 & 17 \end{array}\right)$
2. Determine if the following matrix is in REF, RREF, or neither: $\left(\begin{array}{ccc} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right)$
3. Find the solution to the system represented by the RREF matrix $\left(\begin{array}{ccc|c} 1 & 0 & 2 & 4 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right)$. Express the solution in terms of a free variable z .

Solutions

1. Transform the following augmented matrix into Reduced Row Echelon Form: $\left(\begin{array}{cc|c} 1 & 3 & 7 \\ 2 & 8 & 17 \end{array}\right)$
- Step 1: Eliminate the entry below the first pivot. $R_2 \leftarrow R_2 - 2R_1$ gives $\left(\begin{array}{cc|c} 1 & 3 & 7 \\ 0 & 2 & 3 \end{array}\right)$.
 - Step 2: Scale the second row to make the pivot one. $R_2 \leftarrow 0.5R_2$ gives $\left(\begin{array}{cc|c} 1 & 3 & 7 \\ 0 & 1 & 1.5 \end{array}\right)$.
 - Step 3: Eliminate the entry above the second pivot. $R_1 \leftarrow R_1 - 3R_2$ gives $\left(\begin{array}{cc|c} 1 & 0 & 2.5 \\ 0 & 1 & 1.5 \end{array}\right)$.

Answer: $\begin{pmatrix} 1 & 0 & 2.5 \\ 0 & 1 & 1.5 \end{pmatrix}$

2. Determine if the following matrix is in REF, RREF, or neither: $\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{pmatrix}$
- Step 1: Check REF conditions. All zero rows are at the bottom. The leading entry of row 2 is to the right of row 1. All entries below pivots are zero. It is in REF.
 - Step 2: Check RREF conditions. All pivots are one. The pivot in column 1 is the only non-zero entry in column 1. The pivot in column 2 is the only non-zero entry in column 2.

Answer: RREF

3. Find the solution to the system represented by the RREF matrix $\left(\begin{array}{ccc|c} 1 & 0 & 2 & 4 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right)$. Express the solution in terms of a free variable z .

- Step 1: Identify pivots. Pivots are in columns 1 and 2. Column 3 has no pivot, so z is a free variable.
- Step 2: Write the equations from the rows. Row 1: $x + 2z = 4$. Row 2: $y - z = 3$.
- Step 3: Solve for pivot variables. $x = 4 - 2z$ and $y = 3 + z$.

Answer: $x = 4 - 2z$, $y = 3 + z$, z is free