Gaussian elimination II — harder systems, pivots

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Practice problems

1. Solve the following system using Gaussian elimination: $x+2y-z=3$, $y+4z=2$, $2x+5y+2z=10$.
2. Solve the system: $0x+y+z=2$, $x+2y+z=4$, $2x+7y+z=19$.
3. Solve the system: $x+y+z=6$, $2x+4y-2z=2$, $3x+5y+0z=18$.

Solutions

1. Solve the following system using Gaussian elimination: $x+2y-z=3$, $y+4z=2$, $2x+5y+2z=10$.
   - Write the augmented matrix: $[[1,2,-1,3],[0,1,4,2],[2,5,2,10]]$.
   - Eliminate x from row 3: $R_3 \leftarrow R_3 - 2R_1$ gives $[[1,2,-1,3],[0,1,4,2],[0,1,4,4]]$.
   - Eliminate y from row 3: $R_3 \leftarrow R_3 - R_2$ gives $[[1,2,-1,3],[0,1,4,2],[0,0,0,2]]$.
   - Observe the third row: $0x+0y+0z=2$, which is impossible.

   Answer: No solution (Inconsistent system)

2. Solve the system: $0x+y+z=2$, $x+2y+z=4$, $2x+7y+z=19$.
   - Swap R_1 and R_2 to get a pivot in the first column: $[[1,2,1,4],[0,1,1,2],[2,7,1,19]]$.
   - Eliminate x from row 3: $R_3 \leftarrow R_3 - 2R_1$ gives $[[1,2,1,4],[0,1,1,2],[0,3,-1,11]]$.
   - Eliminate y from row 3: $R_3 \leftarrow R_3 - 3R_2$ gives $[[1,2,1,4],[0,1,1,2],[0,0,-4,5]]$.
   - Back-substitute: $-4z=5 \implies z=-1.25$.
   - Substitute z into R_2: $y-1.25=2 \implies y=3.25$.
   - Substitute y,z into R_1: $x+6.5-1.25=4 \implies x=-1.25$.

   Answer: x = -1.25, y = 3.25, z = -1.25

3. Solve the system: $x+y+z=6$, $2x+4y-2z=2$, $3x+5y+0z=18$.
   - Augmented matrix: $[[1,1,1,6],[2,4,-2,2],[3,5,0,18]]$.
   - Clear Col 1: $R_2 \leftarrow R_2-2R_1$ and $R_3 \leftarrow R_3-3R_1$ gives $[[1,1,1,6],[0,2,-4,-10],[0,2,-3,0]]$.
   - Clear Col 2: $R_3 \leftarrow R_3 - R_2$ gives $[[1,1,1,6],[0,2,-4,-10],[0,0,1,10]]$.
   - Back-substitute: $z=10$.
   - Substitute z into R_2: $2y-40=-10 \implies 2y=30 \implies y=15$.
   - Substitute y,z into R_1: $x+15+10=6 \implies x+25=6 \implies x=-19$.

   Answer: x = -19, y = 15, z = 10
