Gaussian elimination I — the algorithm

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Practice problems

1. Solve the system using Gaussian elimination: $2x + 3y = 8$ and $x - y = -3$.
2. Transform the following system into upper triangular form using row operations: $x + y + z = 6$, $2x - y + z = 3$, and $x + 2y - z = 2$.
3. Solve the system from the previous problem using back-substitution.

Solutions

1. Solve the system using Gaussian elimination: $2x + 3y = 8$ and $x - y = -3$.

   - Write the augmented matrix: $\left(\begin{array}{cc|c} 2 & 3 & 8 \\ 1 & -1 & -3 \end{array}\right)$
   - Swap rows to get a pivot of 1: $\left(\begin{array}{cc|c} 1 & -1 & -3 \\ 2 & 3 & 8 \end{array}\right)$
   - Eliminate below pivot: $R_2 \leftarrow R_2 - 2R_1$ gives $\left(\begin{array}{cc|c} 1 & -1 & -3 \\ 0 & 5 & 14 \end{array}\right)$
   - Solve for y: $5y = 14 \implies y = 2.8$
   - Back-substitute for x: $x - 2.8 = -3 \implies x = -0.2$

   Answer: x = -0.2, y = 2.8

2. Transform the following system into upper triangular form using row operations: $x + y + z = 6$, $2x - y + z = 3$, and $x + 2y - z = 2$.

   - Augmented matrix: $\left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 2 & -1 & 1 & 3 \\ 1 & 2 & -1 & 2 \end{array}\right)$
   - Clear first column: $R_2 \leftarrow R_2 - 2R_1$ and $R_3 \leftarrow R_3 - R_1$ gives $\left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -3 & -1 & -9 \\ 0 & 1 & -2 & -4 \end{array}\right)$
   - Swap R_2 and R_3 for easier pivot: $\left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & -2 & -4 \\ 0 & -3 & -1 & -9 \end{array}\right)$
   - Clear second column: $R_3 \leftarrow R_3 + 3R_2$ gives $\left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & -2 & -4 \\ 0 & 0 & -7 & -21 \end{array}\right)$

   Answer: \left(\beginarrayccc—c 1 & 1 & 1 & 6 \\ 0 & 1 & -2 & -4 \\ 0 & 0 & -7 & -21 \endarray \right)

3. Solve the system from the previous problem using back-substitution.

   - From the last row: $-7z = -21 \implies z = 3$
   - From the second row: $y - 2(3) = -4 \implies y - 6 = -4 \implies y = 2$
   - From the first row: $x + 2 + 3 = 6 \implies x + 5 = 6 \implies x = 1$

   Answer: x = 1, y = 2, z = 3
