When do solutions exist? Geometric intuition

Practice problems

1. Determine the number of solutions for the system: $3x - 2y = 5$ and $6x - 4y = 10$. Explain your reasoning geometrically.
2. Consider the system $x + y = 2$ and $x + y = 5$. How many solutions exist, and why?
3. In 3D, if three planes are all parallel to each other and distinct, how many solutions does the system have?

Solutions

1. Determine the number of solutions for the system: $3x - 2y = 5$ and $6x - 4y = 10$. Explain your reasoning geometrically.

   - Observe the coefficients of the first equation: 3 and -2.
   - Observe the coefficients of the second equation: 6 and -4.
   - Notice that $6 = 2 \times 3$ and $-4 = 2 \times (-2)$.
   - Check the constants: $10 = 2 \times 5$.
   - Since the entire second equation is exactly twice the first, the lines are coincident.

   Answer: Infinitely many solutions

2. Consider the system $x + y = 2$ and $x + y = 5$. How many solutions exist, and why?

   - The normal vectors for both lines are $[1, 1]$.
   - Since the normal vectors are identical, the lines are parallel.
   - The constants 2 and 5 are different, so the lines are distinct.
   - Parallel distinct lines never intersect.

   Answer: No solution

3. In 3D, if three planes are all parallel to each other and distinct, how many solutions does the system have?

   - A solution must be a point that lies on all three planes simultaneously.
   - If plane 1 and plane 2 are parallel and distinct, they have no points in common.
   - Therefore, there can be no point that lies on all three planes.

   Answer: No solution
