Systems of equations: the row picture and the column picture

Practice problems

1. Given the system $\begin{cases} 3x + 2y = 7 \\ 4x - y = 2 \end{cases}$, write the corresponding vector equation for the column picture.
2. Consider the system $\begin{cases} x + y = 2 \\ 2x + 2y = 4 \end{cases}$. Describe the row picture and the column picture.
3. Find the solution (x, y) for the system $\begin{cases} 2x + y = 5 \\ x - y = 1 \end{cases}$ using the column picture logic.

Solutions

1. Given the system $\begin{cases} 3x + 2y = 7 \\ 4x - y = 2 \end{cases}$, write the corresponding vector equation for the column picture.

   - Identify the coefficients of x as the first column vector: $\mathbf{v}_1 = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$.
   - Identify the coefficients of y as the second column vector: $\mathbf{v}_2 = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$.
   - Identify the constants on the right side as the target vector: $\mathbf{b} = \begin{pmatrix} 7 \\ 2 \end{pmatrix}$.
   - Combine them into the form $x\mathbf{v}_1 + y\mathbf{v}_2 = \mathbf{b}$.

   Answer: x \beginpmatrix 3 \\ 4 \endpmatrix + y \beginpmatrix 2 \\ -1 \endpmatrix = \beginpmatrix 7 \\ 2 \endpmatrix

2. Consider the system $\begin{cases} x + y = 2 \\ 2x + 2y = 4 \end{cases}$. Describe the row picture and the column picture.

   - In the row picture, the second equation is just twice the first, so both equations describe the same line $x + y = 2$.
   - In the column picture, the columns are $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$.
   - The target vector is $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$, which is exactly twice the first column.
   - Since the columns are dependent and the target is in their span, there are infinite solutions.

   Answer: Both rows describe the same line; both columns are the same vector and the target is a multiple of them.

3. Find the solution (x, y) for the system $\begin{cases} 2x + y = 5 \\ x - y = 1 \end{cases}$ using the column picture logic.

   - Write as $x \begin{pmatrix} 2 \\ 1 \end{pmatrix} + y \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$.
   - Try small integers: if $x = 2$, then $2 \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$.
   - We need $\begin{pmatrix} 5 \\ 1 \end{pmatrix} - \begin{pmatrix} 4 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$.
   - This is exactly 1 times the second column vector, so $y = 1$.
   - Check: $2(2) + 1 = 5$ and $2 - 1 = 1$.

   Answer: x = 2, y = 1